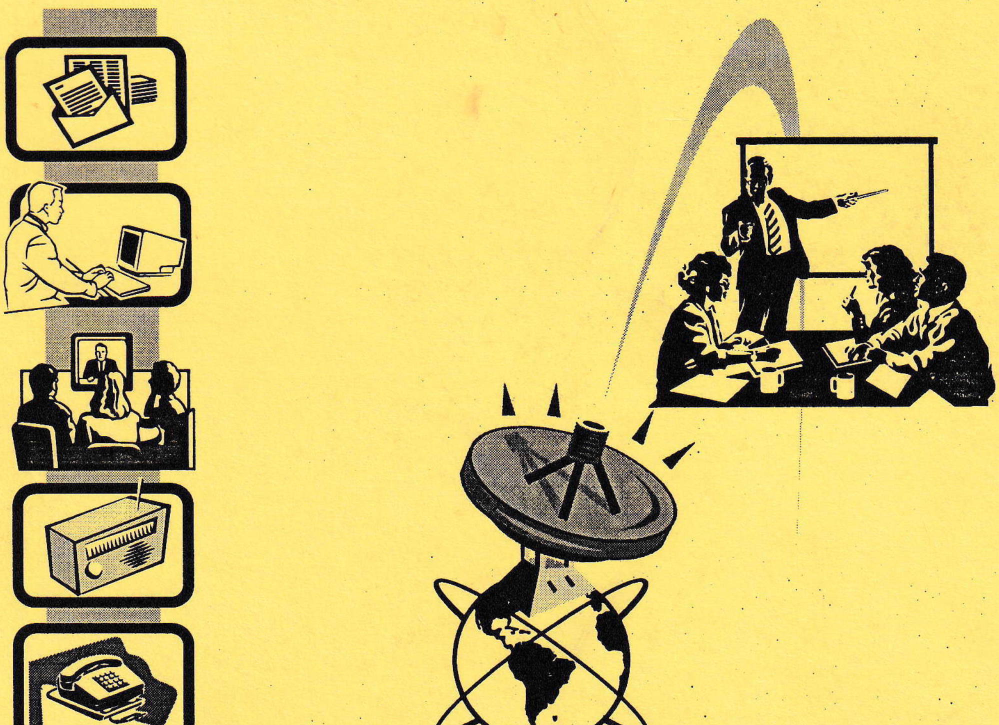
INSTITUTE OF ADULT EDUCATION

OPEN AND DISTANCE LEARNING

MATHEMATICS

TEWW M_2

MODULE 11
Sequences, Series, Rate and Variation



Unit 11:1 Sequence and Series, Arithmetic Progression
Unit 11:2 Geometric Progression
Unit 11:3 Rate and Variation

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MODULE 11
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Unit 11.1 Sequence and Series, Arithmetic Progression

Unit 11.2 Geometric Progression

Unit 11.3 Rate and Variation

Institute of Adult Education ODL Study Materials

CONTENTS

11.0.1 INTRODUCTION TO THE MODULE

In Module 10, you studied relations and functions. I hope you enjoyed it. In this Module 11 you are going to study sequence and series, rate and variations.

11.0.2 OBJECTIVES OF THE MODULE

After completing this module you should be able to:

- Differentiate between sequences and series
- Identify Arithmetic Progressions and Geometric Progressions
- Find the $\mathbf{n}^{th}$ term, sum and mean of an Arithmetic Progression (AP) and Geometric Progressing (GP).
- Calculate simple interest and compound interest respectively.
- Relate quantities of same kind and different kinds in calculations of rate and proportions.
- Apply the knowledge of rates to convert Tanzanian currency into other currencies and vice versa.
- Solve problems involving rates and variations.

UNIT 11.1

SEQUENCES AND SERIES, ARITHMETIC PROGRESSIONS

CONTENTS

11.1.0.1 INTRODUCTION

In the previous unit you learned about data presentation and calculations of central tendency and measurement of data. You arranged data either from the smallest to the biggest or from the biggest to the smallest without conditions. In this unit you are going to learn about **sequences and series**.

The knowledge of sequences and series is important in business because it helps people who deposit money in the bank to know how much interest has been gained after a period of time. The word population can be determined by looking at the population growth rate. Instead of using names or activities we use numbers with the condition governing the second number in the sequence or series. The study about arrangement of numbers in given conditions is known as sequences and series.

This unit covers four sections that are, sequences, arithmetic progression, geometric progression and series.

11.1.0.2 OBJECTIVES

After completing this unit you should be able to:

- identify a sequence and find the missing terms of a sequence;
- find the general term of a sequence;
- define a sequence and series;
- define Arithmetic Progression (AP);
- derive the formulae for Nth term and Summation (Sn) of AP and apply it in solving problems related to AP;
- find the Arithmetic Mean of AP;

11.1.1 SEQUENCE AND SERIES

Sequences

In Mathematics we have patterns of numbers with special characteristics, for example Natural numbers which start with 1,2,3, ... , set of even numbers 2,4, 6 ... , and set of odd numbers 1,3,5,7 .... In any of these patterns it is easy to list all the required numbers because they have special characteristics. But there are some number patterns which need some knowledge so as to be able to list down their elements. These are known as Number sequences, each number in the sequence is called a term.

<u>Example</u>
Look at the numbers in the following pattern. 3, 5, 7, 9, 11 . . .

What is the next number after 11?
In the sequence the first term is 3. and the second term is 5 you can see that each number is two units greater than the number before it (preceding numbers). So the number after 11 should be 13.

The sequence becomes: 3, 5, 7, 9, 11, 13.

<u>Example</u>
What are the next two numbers in the sequence 1, 4, 7, 10 ___, ___ ?
If you observe the above sequence, you can see that each number in the sequence is three units greater than the number before it.

So the next two numbers in the sequence will be obtained by adding 3;
1, 4, 7, 10, 10 + 3, 10 + 3 + 3
Then the sequence becomes, 1, 4, 7, 10, 13, 16;
The required numbers are 13 and 16.

<u>Example</u>
What are the next two numbers in the following sequence -6, -3, 0, ___ , ___ ?

<u>Solution</u>:
Here also the difference between two consecutive numbers is 3. To get the required number we add 3 to the last number in the sequence. -6, -3, 0 3, 6.
The required numbers are 3 and 6.

<u>Example</u>
What are the missing numbers in the following sequence: 1, 2, 3, 5, 8, 13, ____, ____?

<u>Solution</u>
If you look at the sequence, suppose you decide to subtract two adjacent numbers $2 - 1 = 1$; $3 - 2 = 1$
but $5 - 3 = 2$; $8 - 5 = 3$

So there is no common difference between two consecutive terms, therefore difference will not help in getting the two terms.

Suppose we decide to add up two consecutive terms.

$$1 + 2 = 3$$
$$2 + 3 = 5$$
$$3 + 5 = 8$$
$$5 + 8 = 13$$

You can see that the pattern above starts after adding the first two terms and the following terms are obtained by adding the consecutive terms.

Therefore missing terms will be obtained as follows:
1, 2, 2 + 1, 3 + 2, 5 + 3, 8 + 5, 13 + 8, 21 + 13

Then your sequence will look like this 2, 2, 3, 5, 8, 13, 21, 34.
The missing terms are 21, 34.

The General term of a Sequence

In sequences each number is called a term, like in sets each number is called an element.
Given the following sequence: 1, 3, 5, 7 **...**
1 is the first term, the second term is 3, and the third term is 5 and so on.

Let; $A_1 = 1$

$A_2 = 3$

$A_3 = 5$

$A_4 = 7$

$A_5 = 9$

•

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•

$A_n = n^{th}$ term

If you use a set of even numbers. 2, 4, 6, 8, 10 **...**

A_1	A_2	A_3	A_4. . .A_n		
	↓	↓	↓	↓	↓
2	4	6	8		n^{th} term.

Let us look more closely at the sequence of even numbers.

There is a relationship between the number indicated on the A and the value of terms in the sequence of even numbers. Let us circle on A and see how that number is used to get the number in the sequence.

$A_1 = 2 \times 1 = 2$

$A_2 = 2 \times 2 = 4$

$A_3 = 2 \times 3 = 6$

$A_4 = 2 \times 4 = 8$

$A_n = 2 \times n = 2n.$

If you look at the numbers which are circled on A and those multiplied by 2 at your right hand, you can see that the number in the sequence depends on the number indicated on A.

$\therefore A_{10} = 2 \times 10 = 20$

$A_{20} = 2 \times 20 = 40$

$A_n = 2 \times n = 2n.$

Therefore for even number $A_n = 2n$.
$A_n = 2n$ for even numbers is called a **General term** of the sequence.

The general term (formula) helps to determine the required value of the term in the sequence without listing all terms up to that term.

Find the general term of the given sequence. 1, 3, 5, 7, . . .

<u>Solution</u>

Look carefully at your sequence and find the relationship between the position of your term and value. List all your sequences as follows.

$A_1 = 1$
$A_2 = 3$
$A_3 = 5$
$A_4 = 7$
$A_5 = 9$
.
.
.
$A_n = ?$

Look at A_n, this can be related as follows:

$A_1 = 1$

But: $A_2 = 3 = 2 + 1$

But $A_3 = 5 = 3 + 2$

Also $A_4 = 7 = 4 + 3$

This can be rewritten as:

$A_1 = 1 + 1 - 1 = 1$

$A_2 = 2 + 2 - 1 = 3$

$A_3 = 3 + 3 - 1 = 5$

$A_4 = 4 + 4 - 1 = 7$

$A_n = n + n - 1 = 2n - 1$

Therefore $A_n = 2n - 1$
The general term; $A_n = 2n - 1$.
This is the general term for odd numbers.

<u>Example</u>

The following are general terms of certain sequences. Write down the first four terms in each sequence.

(i) $A_n = n + 2$
(ii) $A_n = n^2$
(iii) $A_n = \frac{1}{n}$

Solution

(i) Write down the given formula.

$A_n = n + 2.$

To get the sequence, evaluate your solution by using the number (subscript) indicated on A.

A_1: instead of n now it is indicated as 1; therefore put 1 instead of n in your formulae and so on.

$$A_n = n + 2$$

$$A_1 = 1 + 2 = 3$$

$$A_2 = 2 + 2 = 4$$

$$A_3 = 3 + 2 = 5$$

$$A_4 = 4 + 2 = 6.$$

(ii) $A_n = n^2$ use the same procedure as in (1) above

$$A_1 = 1^2 = 1$$

$$A_2 = 2^2 = 4$$

$$A_3 = 3^2 = 9$$

$$A_4 = 4^2 = 16$$

(iii) $A_n = \frac{1}{n}$

$$A_2 = \frac{1}{1} = 1$$

$$A_2 = \frac{1}{2} = \frac{1}{2}$$

$$A_3 = \frac{1}{3} = \frac{1}{3}$$

$$A_4 = \frac{1}{4} = \frac{1}{4}$$

SELF-CHECK EXERCISE 1

Answer the following questions

1. Write down the next three numbers in each of the following sequences

   (I) 3, 7, 11, 15 .....

   (ii) 1, 8, 27, 64 .....

   (iii) 4, 2, 1, $\frac{1}{2}$ .....

   (iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}$, ...

   (iv) 1, 3, 7, 13 .....

   (vi) 2, 5, 10, 17, 26 .....

   (vii) 5, 7, 10, 14 .....

2. Write down the first six terms of the sequence given by the following general terms

   (i) $A_n = 2n + 1$

   (ii) $A_n = 5n - 2$

   (iii) $A_n = 2n^2 + 3$

   (iv) A_n = **Error!**

   (v) $A_n = n^3 - 1$

3. Find the general term of the following sequences.

   (i) 4, 7, 10, 13 . . . .

   (ii) 5, 7, 9, 11 . . . .

   (iii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ . . . .

   (iv) 0, 7, 26, 63 . . . .

Compare your answers with those given at the end of the unit.

SUMMARY

1. A set of numbers given in a particular pattern is called a sequence.
2. From any sequence, you can develop a general term A_n.
3. The general term gives the value of any term in a sequence without necessarily listing other terms.

11.1.2 SERIES

In section one we discussed sequences. In this section you will learn about series.

Series

If your listed numbers are added together you no longer call it a sequence but series. Therefore you can define series as the sum of all terms in a sequence. You have two common types of series, **FINITE** and **INFINITE SERIES**.

You study series so that you can be able to predict or determine the end of some phenomena which take place in our daily life naturally or by man's plan. For example the number of **AIDS** cases in a particular area after a given time interval can be determined by looking at the multiplication rate and make summation of the series. The number of trees cut in a particular area can be determined by looking at the harvesting rate in a fixed time and you can predict the effect of that phenomenon. Now let us study the two types of our series and how we can use them..

Finite and Infinite Series

If a series ends after a known number of terms, it is said to be a finite series. For example $2 + 4 + 8 + 16 + 32$.

This type of series is **finite series** and ends after the fifth term. The series has five terms starting with 2 and ending with 32.
The series $2 + 4 + 8 + 16 + 32 +$ **...is** an **infinite series.** The three dots after 32+ show that the series has no end point (term). So you cannot sum up all the terms of the series.

The general term of series

The sum of all n^{th} terms of the series is usually denoted by S_n;
The sum of $3 + 4 + 5 + 6 + 7 + ... + n + 2$ can be written as

$$S_n = 3 + 4 + 5 + 6 + 7 + ... + (n + 2).$$

Example

The terms of a certain sequence are 0, 3, 6, 9, ...

(i) Write down the series corresponding to the sequence
(ii) Find the general term.
(iii) Find the sum of the first four terms.

Solution

(i) We write the same numbers with summation
$0 + 3 + 6 + 9 + ...$

(ii) The general term.
$A_1 = 0 = 3(1 - 1)$
$A_2 = 3 = 3(2 - 1)$
$A_3 = 6 = 3(3 - 1)$
$A_4 = 9 = 3(4 - 1)$
$A_n = n^{th}$ term $= 3(n - 1)$
The general term is $3n - 3$

(iii) The sum of the first four terms is

$S_4 = 0 + 3 + 6 + 9$

$= 18$

Therefore; $S_4 = 18$

<u>Example</u>

Consider the given sequence: 8, 6, 4, 2

(i) Write down the series corresponding to the sequence

(ii) Find the general term of series.

(iii) Find the sum of the first nine terms.

<u>Solution</u>

(i) The series corresponding to the given sequence is:

$A_n = 8 + 6 + 4 + 2 + \ldots$

(ii) The general term.

$A_1 = 8 = 10 - 2 \times 1$

$A_2 = 6 = 10 - 2 \times 2$

$A_3 = 4 = 10 - 2 \times 3$

$A_4 = 2 = 10 - 2 \times 4$

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$A_n = n^{th}$ term $= 10 - 2n$.

The general term; $A_n = 10 - 2n$.

(iii) The sum of the first nine terms is

$S_9 = 8 + 6 + 4 + 2 + 0 + (-2) + (-4) + (-6) + (-8) = 0$

$S_9 = 0$.

SELF-CHECK EXERCISE 2

Answer the following questions

1. For each of the following sequences:-
   (a) Write down the first 6 terms and sum of the first 6 terms of the series corresponding to.
   (b) Find the general term of the series.

   (i) $1, \frac{1}{2}, \frac{1}{3}, \ldots$
   (ii) 6, 10, 14, ...
   (iii) 6, 8, 10, ...
   (iv) 1, 4, 9, ...
   (v) 1, 5, 9, ...

2. The first term of a certain series is $\boldsymbol{a}$, the second term is $\mathbf{2}\boldsymbol{a}$, the third term is $\mathbf{3}\boldsymbol{a}$, Find the
   (i) general term.
   (ii) sum of the first ten terms.

3. The general term of a series is $5n + 1$, find the sum of the first 5 terms.

4. Find the 3rd and 10th term of the series whose general term is $2n^2 1$.

Compare your answers with those given at the end of the unit.

NOTE

1. A series is summation of all terms in a sequence.

2. A finite series is a series with a known number of terms.

3. An infinite series is a series having an unknown number of terms.

4. A general term for a series is a formula which when used gives all terms of that series:
   For even numbers: $A_n = 2n$
   For odd numbers: $A_n = 2n\ 1$
   For natural numbers: $A_n = n$

11.1.3 ARITHMETIC PROGRESSION (A.P) AND SIMPLE INTEREST

Arithmetic Mean

In our previous discussion, you have seen how sequences form series. The series can be grouped further with those with special characteristics forming one group. This is because the process of finding general terms and summation of terms can be time consuming. Consider the summation of the first 100 terms of:

(i) $2 + 4 + 6 + 8 + ...$

or

(ii) $1 + 3 + 5 + 7 + 9 + ...$

or

(iii) $-6 + (-3) + 0 + 3 + 6 + ...$

This can be a boring and time consuming exercise so we have to study common characteristic of a given series so that we can develop a quick way of finding n^{th} term and summation of the first n^{th} terms.

Consider our first series above:

(i) $2 + 4 + 6 + 8 + ...$

$A_1 = 2$

$A_2 = 4$

$A_3 = 6$

$A_4 = 8$

Therefore: $A_2 - A_1 = 2$

$A_3 - A_2 = 2$

$A_4 - A_3 = 2$

The difference between two consecutive terms is 2; Look again at the following series.

$1 + 2 + 3 + 4 + 5 + ...$

$A_1 = 1$

$A_2 = 2$

$A_3 = 3$

$A_4 = 4$

Then, $A_2 - A_1 = 1$

$A_3 - A_2 = 1$

$A_4 - A_3 = 1$

The common difference between two consecutive terms is 1, similarly $-6 + (-3) + 0 + 3$ the common difference between two terms is 3.

After observing the three examples you can conclude that there is a special series whose terms are obtained by adding a fixed number to the number before it. This series has a special name called Arithmetic Series or Arithmetic Progression (AP).

Any series having a fixed difference between two consecutive terms is known as Arithmetic Progression. The fixed number which is the difference between any two consecutive terms is called the common difference (d).

Example

Which of the following series is Arithmetic Progressions (AP)? Write down the common difference (d) for those which are Arithmetic Progressions.

(i) $0 + 3 + 6 + 9 + ...$
(ii) 1 + 1**Error!** + 2 + 2**Error!** + ...
(iii) $1 + 3 + 7 + 13 + ...$
(iv) $2 + 4 + 6 + 10 + 16...$
(v) 5, 10, 15, 20...

Solution:
(i) Is an Arithmetic Progression, The common difference(d) = 3
(ii) Is an Arithmetic Progression, The common difference(d) = $\frac{1}{2}$
(iii) Is not AP
(iv) Is not AP
(iv) Is not AP, it is just a sequence, because there is no summation sign in between.

The general term of an Arithmetic Progression

We have defined arithmetic progression as a series having common differences between two consecutive terms.

If $A_1 = 2$ and the common difference is 2
Then: $A_1 = 2$
$A_2 = 2 + 2$
$A_3 = 2 + 2 + 2$
$A_4 = 2 + 2 + 2 + 2$
$A_5 = 2 + 2 + 2 + 2 + 2$

The last statement can be summarised as
$A_5 = 2 + (5 - 1)2$
and $A_n = 2 + (n - 1)2$
$A_n = 2 + 2n - 2$
$= 2n.$

The general term of AP is denoted by A_n and for the above AP, $\mathbf{A_n = 2n}$ in general if d is the common difference; we have.

$A_1 = A_1$
$A_2 = A_1 + d$
$A_3 = A_2 + d = (A_1 + d) + d = A_1 + 2d$
$A_4 = A_3 + d = (A_1 + 2d) + d = A_1 + 3d$
$A_5 = A_4 + d = (A_1 + 3d) + d = A_1 + 4d$
$A_n = A_1 + (n - 1)d$

Therefore

$$\mathbf{A_n = A_1 + (n-1)d}$$

d = common difference
A_1 = first term
n = is the position of terms of AP whose values are being found.

<u>Example</u>
The first term of Arithmetic Progression is -1 and the common difference is 2, find the:

(i) fourth term
(ii) general term

<u>Solution</u>

(i) Given $A_1 = -1$, $d = 2$, $n = 4$
Using the formulae
$A_n = A_1 + (n-1)d$
$A_4 = -1 + (4-1)2$
$= -1 + 3\times2$
$= -1 + 6$
$= 5$
$\mathbf{A_4 = 5}$

(ii) The general term A_n:
$A_n = A_1 + (n-1)d$ but; $A_1 = -1$
$d = 2$

$A_n = -1 + (n-1)2$
$= -1 + 2n-2$
$\mathbf{A_n = 2n-3}$

<u>Example</u> 5 + 8 + 11 + 14
Find (i) general term of AP
(ii) A_{15}. (15^{th} term)

<u>Solution</u>

(i) $A_1 = 5$
The common difference(d) is $A_2 - A_1 = 8-5=3$.
using the formulae
$A_n = A_1 + (n-1)d$
$= 5 + (n-1)3$
$= 5 + 3n-3$
$\mathbf{A_n = 2 + 3n}$

(ii) A_{15} or 15^{th} term,
using the formula:
$A_n = 2 + 3n$ but $n = 15$
$A_{15} = 2 + 3\times15$
$= 47$

Example

$A_n = 2$ is an AP

(i) What is the common difference d.

(ii) Write down the first 10 terms.

Solution:

(i) $A_1 = 2$

$A_2 = 2$

common difference (d) $= A_2 - A_1 = 32 = 0$

$d = 0$

(ii) The first 10 terms are

2 , 2 , 2 , 2 , 2 , 2 , 2 , 2 , 2 , 2.

Example

The third term of Arithmetic Progression is 7 and the fifth term is 13. Find.

(i) The common difference

(ii) The first term

(iii) The n^{th} term.

Solution

given $A_3 = 7$

$A_5 = 13$

using the formula;

$A_n = A_1 + (n1)d$

$A_3 = A_1 + (31)d \Rightarrow 7 = A_1 + 2d$

$A_5 = A_1 + (51)d \Rightarrow 13 = A_1 + 4d$

> These are simultaneous equations with A_1 and d unknowns we must solve for d and A_1.

Solving the simultaneous equations:

$13 = A_1 + 4d$

$-7 = A_1 + 2d$

$6 = 2d \Rightarrow d = 3$

(i) **$d = 3$ common difference**

then $7 = A_1 + 2 \times 3$

$7 = A_1 + 6$

$A_1 = 1$

(ii) **The first term $A_1 = 1$**

(iii) using the formula:

$A_n = A_1 + (n1)d$ $\quad A_1 = 1$

$d = 3$

$An = 1 + (n1)3$

$= 1 + 3n3$

$= 3n2$

The general term $A_n = 3n2$

The sum of the first n terms of AP

When we have Arithmetic Progression, we sometimes need to sum all the values of the n^{th} terms.

For example given an AP: $2 + 4 + 6 + 8 + 10$
We can easily add these four terms: $2 + 4 + 6 + 8 + 10 = 30$
But AP can have many terms for example $2 + 4 + 6 + 10 + ... + 100 + 102 + 104$.

If you are asked to sum up all these terms you need a lot of time and patience to do it. From this fact we need to develop a short way of summing up the terms easily.

Sum up the first 30 terms of:-

(i) $2 + 4 + 6 + ...$
(ii) $1 + 2 + 3 + ...$
(iii) $4 + 2 + 0 + ...$

I think you have experienced how tough it is and how much time will be required for doing it.

Now, consider the following AP; if S_n stands for summation;
$S_n = 2 + 4 + 6 + 8 + 10$, let this be equation ① we can rearrange our AP and write into another from $S_n = 10 + 8 + 6 + 4 + 2$. Let this be equation ② because addition is commutative.

If we add up the two AP vertically, as follows we will simply have;

$$S_n = 2 + 4 + 6 + 8 + 10$$
$$+ \underline{S_n = 10 + 8 + 6 + 4 + 2}$$
$$2S_n = 12+12+12+12+12$$
$$2S_n = 12 \times 5$$

S_n = **Error!**but 5 is the number of terms in our AP, if we divide by 2 each side.
= 30 This is equal to what we got in an earlier example when we added term by term.

This can be generalised by using AP with the first terms A_1 and A_n as the last term.

$$S_n = A_1 + (A_1 + d) + A_1 + 2d + A_1 + 3d + \ldots + A_n$$
$$+ S_n = A_n + (A_n \tilde{} d) + A_n \tilde{} 2d + (A_n \tilde{} 3d) \ldots + A_1$$
$$2S_n = (A_1 + A_n) + (A_1 + An) \ldots\ldots (A_1 + A_n)$$
$$2S_n = n(A_1 + A_n) \text{ divide by 2 each side.}$$
S_n = **Error!**$(A_1 + A_n)$

S_n = **Error!** (A_1+A_n)

A_1 = first term
A_n = last term
n = number of terms

but $A_n = A_1 + (\bar{n}1)d$

Therefore S_n = **Error!**$\left(A_1 + A_1 + (\bar{n}1)d\right)$

= **Error!** $\left[2A_1 + (\bar{n}1)d\right]$

Therefore the sum of the first n terms of AP with A_1 and common difference(d) is given by.

S_n = **Error!** [$2A_1$+

Example
Find the sum of the first 6 terms of

$2 + 3 + 4 + 5 + \ldots$

Solution

Using the formulae

S_n = **Error!** $\left[2A_1 + (n-1)d\right]$

$A_1 = 2$

$d = 1$

$n = 6$

Therefore $S_n = \frac{6}{2}\left[2 \times 2 + (6-1) \times 1\right]$

$= 3\left[4 + 5\right]$

$\mathbf{S_n = 27}$

Example
The n^{th} term of an AP is given by $3\bar{n}2$. Find the sum of the first 50 terms.

Solution
You can find the solution of this question by two approaches; first you can find the

$A_1 = 3 \times \bar{1}2 = 1$

$A_{50} = 3\times5\bar{0}2 = 148$

Then you use the formulae

S_n = **Error!**($A_1 + A_n$)

= **Error!**(1 + 148)

$\mathbf{S_n = 3725}$

SELF-CHECK EXERCISE 3

You are through with Section Two, now do the following exercise.

(1) Which of the following series are Arithmetic Progression AP. Identify them and find the common difference (d).

(i) $1 + 3 + 5 + 7 + 7 + ...$

(ii) $1 - 1 + 3 - 4 + 5 + ...$

(iii) $1 + 1 + 1 + 1 + 1 + ...$

(iv) $0 - 1 - 2 - 3 + ...$

(2) For the following Arithmetic Progressions, find the first six terms for given values and general terms.

(i) $A_1 = 3$ $d = 1$

(ii) $A_1 = -2$ $d = -2$

(iii) $A_1 = 5$ $d = 3$

(3) Find the general term of an Arithmetic Progression whose first term is $(a + 2)$ and the common difference is $\boldsymbol{a}$.

(4) The third term of an Arithmetic Progression is 0 and the common difference is -2; find.

(i) The first term

(ii) the 300^{th} term

(iii) the general term

(5) Juma is planning to plant 100 trees in the tree planting campaign. He decides to start by planting 20 trees in the first month. How many months will it take him if he plants 5 trees monthly after planting the 20 trees in the first month of the campaign ?

(6) Find the sum of the indicated $\mathbf{n}^{\mathbf{th}}$ term of Arithmetic Progression.

(i) $A_1 = 4$ $A_{20} = 40$ $n = 20$

(ii) $A_1 = -2$ $d = -\frac{1}{2}$ $n = 10$

(iii) $A_1 = -12$ $A_8 = -26$ $n = 8$

(7) The sixth term of AP is 56 and tenth term is 72 find A_1, $\boldsymbol{d}$ and $\mathbf{S}_{\boldsymbol{n}}$ for $\boldsymbol{n} = 10$.

(8) A bicycle loses its value by 5,500 shillings annually. After how many years will its value be zero, if the present value is 50,000 shillings?

(9) Find the sum of the integers between 1 and 100 which are divisible by 3.

Compare your answers with those given at the end of the unit.

Simple Interest

In the last four sections you learned about sequences, series, and arithmetic progression. One area of application of this knowledge is saving money in the banks. The oldest way of saving money is keeping it at home.

When we deposit our money into banks there is a profit generated according to the amount of money invested and the policy chosen. This process of profit generation uses either the principle of Arithmetic Progression or Geometric Progression.

In the principle of Arithmetic Progression the profit generated is Simple Interest. The Compound Interest is generated by the principle of Geometric Progression. The choice of the mode of saving depends on the individual and on the bank policy.

Simple interest depends on the amount of money deposited (Principal), the length of time the money stayed in the bank (T) and the rate of interest per one hundred shillings (R)

$$\text{Simple Interest } I = \frac{PRT}{100}$$

P = Principal: The money you deposit into the bank at the beginning of the investment.

R = The profit gained per hundred shillings per unit time. This may be one year, three months, one month or six months (In this unit, R will be for one year).

T = The length of time the money stayed (was deposited) in the bank. (T)

I = Interest (profit gained after time (T)

Example

The IAE Centre Coordinator of Kimanye decided to deposit 10,000 shillings in a bank for a period of five years, and the bank is gives a rate of 10%. How much money will she collect at the end of the five years?

$$I = \frac{PRT}{100} \qquad P = 10,000$$

$R = 10$

$T = 5$

$$\text{Therefore } I = \frac{10000 \times 10 \times 5}{100} = 5,000 \text{ shillings}$$

So after five years she will collect the principal plus interest

$Ps = P + I = 10,000 + 5,000 = 15,000$ shillings.

She will collect 15,000 shillings.

If we use Arithmetic Progression, this can be calculated as follows:

The interest for one year is taken to be the common difference (d).

$$I = \frac{PRT}{100} \qquad T = 1$$

$R = 10$
$P = 10,000$

$$I = \frac{10000 \times 10 \times 1}{100} = 1,000 = d$$

Use the formula
$P_n = P + (\tilde{n}1)d$ $d = 1,000$
$P = 10,000$
$n = k + 1$
k = is the number of years the money stays in the banks.

$P = 10,000 + (6-1)1,000$
$= 10,000 + 5,000$
$= 15,000$ shillings
This is the money to be collected after 5 years.

But for easy calculation all this has been simplified and called Simple Interest.

$$P_T = P + \frac{PRT}{100}$$

P_T = Money to be collected after time T

SELF CHECK EXERCISE 4

Answer all questions carefully

Find the simple interest on:

(1) shs 200,000 invested for 2 years at 6% p.a.

(2) shs 150,000 invested for 5 years at 8% p.a.

(3) shs 650,000 invested for 4 years at 6% p.a.

(4) What annual rate of simple interest is necessary to give interest of shs 416000 on a principal of shs 800000 invested for 8 years?

(5) What sum of money earns shs 312000 simple interest if invested for 8 years at 13%.

Compare your answers with those given at the end of the unit.

11.1.4 SUMMARY OF THE WHOLE UNIT

After completing the unit, here are the main points to remember.

1. A sequence is a flow of numbers given in a particular pattern. The formulae used to get the values of the terms using a set of natural numbers is called general term.

2. A series is defined as a summation of all terms in a sequence.

3. (i) Finite series have a known number of terms.
   (ii) Infinite series have no limits on their terms.

4. Arithmetic Progression (AP) is any series having fixed differences between consecutive terms. The fixed number is known as a common difference(d).

5. The n^{th} and sum of the first n terms of Arithmetic Progression (AP) are found by using the following formulae.

   $A_n = A_1+(n-1)d$
   A_n is equal to the n^{th} term.

   $$S_n = \frac{n}{2}\left[2A_1+(n-1)d\right]$$

   Where:
   A_1 is the first term
   d is the common difference
   n is the number of terms

6. The Arithmetic Mean is the average between two terms in AP and is given by $M = \dfrac{a+b}{2}$.

11.1.5 POST TEST OF THE WHOLE UNIT

After you have covered the whole unit successfully, here is a test to check your understanding of the whole unit before you do your Assignment for submission.

1. What are the missing two numbers in the following sequences?
   (i) 1, -1, 1, -1,_ , _
   (ii) 1, 3, 7, 13, _ , _
   (iii) 1,2,4,7, _ , _

2. The sixth term of AP is 56, the tenth term is 72. Find the first term and common difference.

3. The first term of the AP is 2, the common difference is $\frac{3}{2}$ and $S_n = 72$. Find n and A_n

4. Find the sum of the AP for which $n = 10$, first term is 4 and common difference is -5.

5. Find the arithmetic mean of $42a$ and $52 + 3a$

6. The first term of GP is -2, the common ratio r = 2. Find the sum of the first six terms.

7. Given 10 + 1 + 0.1 + ... is a GP, find the eighth term of the series.

8. John invested 12,000/= in a bank at a 9% rate compounded annually. What will his profit be after 4 years?

9. If 40,000 shs. together with simple interest for 2 years amounts to 50,000 shs. Find the rate per annum (per year).

10. How much money will be collected after four years if 10,000 shs. was invested at 10% compounded annually.

Compare your answers with those given at the end of the unit.

11.1.6 TUTOR MARKED ASSIGNMENT

Now that you have come to the end of the unit, do the following questions in the work book provided and then send the answers to your Tutor for marking and commenting.

Read each question carefully and do it by showing all steps which lead to your solution.

This assignment consists of ten questions, do all of them.

1. Given 3, 10, 17, 24 is a sequence, which is the general term of this sequence.

   (10 Marks)

2. The general term of a sequence is given by $A_n = 3 + (-1)^n$
   List the first five terms of the sequence.

   (10 Marks)

3. Find the missing terms in the following sequence.

   (i) $1, \frac{1}{5}, \_\_\_, \frac{1}{125}, \_\_\_$

   (ii) 7, -1, ___, -17, ___

   (10 Marks)

4. Write the general formula for finding even numbers (positive) starting with 2.

   (10 Marks)

5. (i) What is the sum of the first **n** odd natural numbers

   (i) If 5, x, y and 40 are in geometrical progression, find x and y

   (10 Marks)

6. If the third term of an arithmetic progression is 13 and the tenth term is 27, what is the first term; the 20th term, the 100th term? The sum of the first ten terms.

   (10 Marks)

7. (i) In an arithmetic progression, the sum of the first five terms is 30, and the third term is equal to the sum of the first two. Write down the first five terms of the progression.

   (ii) A child wishes to build a triangular pile of toy bricks so as to have I brick at the top row, 2 in the second. 3 in the third and so on. If he has 100 bricks, how many rows can be complete and how many bricks will he remain with?

   (10 Marks)

8. (i) A village has five farmers with 50, 40, 35, 60 and 70 herds of cattle respectively. The grazing area in the village can support up to 500 herds of cattle only. If the number of cattle increase at the rate of 5, 4, 3, 6 and 7 cattle annually respectively, how many years will farmers live in harmony with their herds in the village if the village decides to maintain the 500 herds of cattle only?

How many cattle will they have to sell during the year when cattle exceed 500?

(ii) Find the arithmetic and geometric mean of 4 and 16.

(10 Marks)

9. (i) Sumuni decided to invest 40,000/= in a bank at the rate of 10% compounded interest annually. How much money will he get after 5 years.

(i) If Athumani is planning to buy a fridge after five years at the rate of 200,000 shillings how much money must he invest if the bank is giving profit of 10% compounded annually?

(10 Marks)

10. Two teachers started their employment at the same time Bwire is a primary school teacher, his starting salary was 40,000 shillings and is getting an increment of 3% annually. The increment is in compound interest. Asha is a secondary school teacher, her starting salary was 45,000 shillings, and getting increment of 3% annually in simple interest form. What will be their salaries after 10 years: Who will be receiving a bigger salary than the other, by how much?

(10 Marks)

11.1.7 KEY ANSWERS TO SELF-CHECK EXERCISES AND POST TEST

Self-Check Exercises

Exercise 1

1. (i) 19,23,27
   (ii) 125,216,343
   (iii) $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
   (iv) $\frac{1}{48}, \frac{1}{96}, \frac{1}{192}$
   (v) 21, 23, 43
   (vi) 37, 50, 65
   (vii) 19, 25, 32

2. (i) 3, 5, 7, 9, 11, 13
   (ii) 3, 8, 13, 18, 23, 28
   (iii) 5, 11, 21, 131
   (iv) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}$
   (v) 0, 7, 26, 63, 124, 215

3. (i) $3n + 1$
   (ii) $2n + 3$
   (iii) $\frac{n}{n+1}$
   (iv) $n^3 1$

Exercise 2

1. (a) (i) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ Sum of 6 terms: $2\frac{9}{20}$
   (ii) 6, 10, 14, 18, 22, 26 $S_6 = 96$
   (iii) 6, 8, 10, 12, 14, 16 $S_6 = 66$
   (iv) 1, 4, 9, 16, 25, 36 $S_6 = 91$
   (v) 1, 5, 9, 13, 17, 21 $S_6 = 66$

   (b) (i) $\frac{1}{n}$
   (ii) $4n + 2$
   (iii) $2n + 4$
   (iv) n^2
   (v) $4n3$

2. (i) na
   (ii) $55a$

3. 70

4. (i) 3rd term; 17;
(ii) 10th term 199

Exercise 3

1. (i) Arithmetic progression, $d = 2$
(ii) Not Arithmetic progression
(iii) Arithmetic progression $d = 0$
(iv) Arithmetic progression $d = -1$

2. (i) 3, 4, 5, 6, 7, 8, $A_n = n + 2$
(ii) -2, -4, -6, -8, -10, -12, $A_n = -2n$
(iii) 5, 8, 11, 14, 17, 20, $A_n = 3n + 2$

3. $A_n + 2$

4. (i) $A_1 = 4$
(ii) -594
(iii) $6\ 2n$

5 17 month

6. (i) $S_{20} = 440$
(ii) $S_{10} = -42\frac{1}{2}$
(iii) $S_8 = -152$

7. $A_1 = 36, d = 4$
$S_{10} = 540$

8. After 9 years

9. After 6 years

10. 1683

Exercise 4

1. 24,000/=
2. 72,000/=
3. 156,000/=
4. $6\frac{1}{2}\%$
5. 300,000/=

Post Test Checklist

1 (i) 1, -1
(ii) 21, 31
(iii) 11, 16

2. $A_1 = 36$, $d = 4$

3. $n = 9$
$A_n = 14$

4. -185

5. $47 + a$

6. -126

7 0.000006

8. 4,325.90 shillings

9 12.5%

10 14,641 shillings

UNIT 11:2

GEOMETRIC PROGRESSION

CONTENTS

11.2.0.1 INTRODUCTION

In the previous unit you studied arithmetic progression and how it can be used in our daily lives. However in this unit you will study another type of sequence called Geometric Progression (GP). Geometric Progression is very useful in industries when dealing with production. In statistics it is used when dealing with deaths and decay of insects, animals etc.

So you can see that G.P is very much applicable in the real life of living things.

11.2.0.2 OBJECTIVES

After completing this unit you should be able to:

- Derive the formulae for the n^{th} term and summation S_n of G.P and apply the formulae in solving problems related to G.P
- Find the geometric means of a G.P.
- Derive the formula for compound Interest and use it to calculate compound Interest.

11.2.1 GEOMETRIC PROGRESSION

This type of series is commonly used in calculating compound interest in bank investments.

Look at the following series.

$2 + 4 + 8 + 16 + 32$

What is the relationship between two consecutive terms?

Therefore; $A_2 - A_1 = 4 - 2 = 2$

$A_3 - A_2 = 8 - 4 = 4$ (the difference is not the same).

If you look at the series carefully, the following is true;

$\frac{A_2}{A_1} = \frac{4}{2} = 2$ This means that $A_2 = 2 \times 2 = A_1 \times 2$

$\frac{A_3}{A_2} = \frac{8}{4} = 2$ This means that $A_3 = 2 \times A_2$

$\frac{A_4}{A_3} = \frac{16}{8} = 2$ This means that $A_4 = 2 \times A_3$

So a series is obtained by multiplying the preceding term by a fixed number. From this we can define our geometric progression (GP).

> A Geometric Progression (GP) is a series of numbers in which each number after the first term can be obtained from the preceding one by multiplying it with a fixed number called the Common ratio (*r*)

<u>Example</u>

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{64}$

This is a geometric progression with a common ratio of $\frac{1}{2}$

<u>Example</u>

What is the common ratio of the following GP.

$5 + 10 + 20 + 40 + \ldots + 640$

<u>Solution</u>

$A_1 = 5$

$A_2 = 10$

The common ratio (r) $= \frac{A_2}{A_1} = \frac{10}{5} = 2$

The common ratio (r) is 2

The general term (n^{th} term) of a geometric progression

If you are given the following sequence:
5 + 10 + 20 + 40 + ... and you are asked to find the 200th term it will take a lot of time to list all terms up to the 200 term. But we can develop a simple way by looking at the patterns of the GP.

Let G_n be the series of geometric progression taking our above GP.

$G_1 = 5 = 5\times2^o = G_1\times r^o = 5$

$G_2 = 5\times2 = 5\times2^1 = G_2\times r^1 = 10$

$G_3 = 5\times2\times2 = 5\times2^2 = G_1\times r^2 = 20$

$G_4 = 5\times2\times2\times2 = 5\times2^3 = G_1\times r^3 = 40$

$G_5 = 5\times2\times2\times2\times2 = 5\times2^4 = G_1\times r^4 = 80$

In this example $r = 2$ and $G_1 = 5$.

You will notice that there is a relationship between the numbers indicated on G and the power to which r is raised.

For example $G_5 = 5\times2^4 = 5\times r^4 = G_1r^4$

The number $G_5 = G_1\times r^{\bar{5}1}$

$= G_1r^4$

We can develop a pattern using G_n, G_1 and r only.

$G_1 = G_1\times r^{11} = Gr^o = G_1$

$G_2 = G_1\times r = G_1r^1$

$G_3 = G_2\times r = G_1r^1\times r = G_1r^2$

$G_4 = G_3\times r = G_1r^3\times r = G_1r^4$

$G_5 = G_4r = G_1r^3\times r = G_1r^4$

•

•

•

$G_n = G_{\bar{n}1}\times r = G_1r^{\bar{n}2}r = G_1r^{\bar{n}1}$

> Therefore; n^{th} term of GP is given by $\mathbf{G_n = G_1r^{n-1}}$
> Where G_1 is a first term
> r = common ratio,
> n = number of terms

<u>Example</u>

The first term of GP is 5 and the common ratio (r) is 3 Find the 5th term.

<u>Solution:</u>

Given $G_1 = 5$

$r = 3$

$n = 5$

Using the formulae.

$G_n = G_1r^{\bar{n}1}$

$G_5 = 5\times3^{\bar{5}1}$

$= 5\times81$

$= 405$

$\mathbf{G_5 = 405}$

Example

Given the following information find the first six terms of a geometric progression;

$G_1 = 1, \ r = 2$

Solution:

Using the formulae

$G_n = G_1 r^{n-1}$

$G_1 = 1\times2^{1\times1} = 1$

$G_2 = 1\times2^{2-1} = 2$

$G_3 = 1\times2^{3-1} = 4$

$G_4 = 1\times2^{4-1} = 8$

$G_5 = 1\times2^{5-1} = 16$

$G_6 = 1\times2^{6-1} = 32$

The first six terms of the GP are 1 + 2 + 4 + 8 + 16 + 32.

Example

Find the tenth term of a GP if the fourth term is 8 and seventh term is 16.

Solution

Given; $G_4 = 8$

$G_7 = 16$

but $G_4 = G_1 r^3$.......①

$G_7 = G_1 r^6$........②

from the fact that $G_n = G_1 r^{n-1}$

if you take equation ② divided by equation ①

$$\frac{16}{8} = \frac{G_1 r^6}{G_1 r^{r3}} = r^3$$

$$2 = r^3$$

Therefore $r = \sqrt[3]{2}$

let us find G_1 first.

$G_4 = 8 = G_1 r^3$

$8 = G_1 r^3$ but $r = \sqrt[3]{2} = 2^{1/3}$

$8 = G_1\left(2^{1/3}\right)^3$

$= G_1\, 2^{3/3}$

$8 = G_1 \cdot 2$

$G_1 = \frac{8}{2} = 4.$

Then $G_{10} = G_1(r)^9 = 4\left((2)^{1/3}\right)^9$

$= 4 \cdot 2^{9/3}$

$= 4 \cdot 2^3.$

$= 32$

$\mathbf{G_{10} = 32}$

1. Given the following information find the first six term of a geometric progression.
   (i) $G_1 = 2$ $r = 0.5$
   (ii) $G_1 = 2$ $r = -2$
   (iii) $G_1 = 2$ $r = -0.5$

2. If the 6^{th} term of a GP is 16 and the 7^{th} term is 64, find the first term.

3. A woman accepts a managerial position at a beginning salary of 60,000 shillings and a 10% increase each year for 8 years. What will be her salary during her eighth year of service in the same position?

Compare your answers with those given at the end of the unit.

The sum of the first N terms of a Geometric Progression

Let us denote summation of GP by S_n as we did in Arithmetic Progression.

$$S_n = G_1 + G_2 + G_3 + G_n + \dots + G_n$$

But; $G_n = G_1 r^{n1}$

Therefore our summation becomes

$$S_n = G_1 + G_1 r + G_1 r^2 + G_1 r^3 + G_1 r^n \text{ equation ①}$$

Let us use a trick, so that we can have another series similar to that in equation ①.

If you multiply the first series in equation ① by r you get

$$rS_n = G_1 r + G_1 r^2 + G_1 r^2 + G_1 r^4 + \dots G_1 r^{n1} r \text{ equation ②}$$

Then last term can be simplified by law of exponents

$$G_1 r^{n1} r = G_1 r^{n1+1} = G r^n$$

Our series becomes

$$rS_n = G_1 r + G_1 r^2 + G_1 r^3 + G_1 r^4 + \dots + G_1 r^n \text{---------(3)}$$

If equation (1) is subtracted from equation (3) vertically

$$-\begin{cases} rS_n = G_1 r + G_1 r^2 + G_1 r^3 + G_1 r^4 + \dots + G_1 r^n \\ S_n = G_1 + G_1 r + G_1 r^2 + G_1 r^3 + \dots + G_1 r^n \end{cases}$$

What remains is

$$rS_n - S_n = G_1 r^n G_1$$

put brackets and take out the common terms

$S_n (r1) = G_1(r^n 1)$ dividing by $(r1)$ each side

$$S_n = \frac{G_1(r^n - 1)}{r - 1}$$

but r is not allowed to be 1 i.e. $r \neq 1$

NB: This formula must be used only when r is greater than 1 for simplification of calculation.

<u>Example</u>

Find the sum of the first 5 terms of the following GP.

$$1 + 3 + 9 + 27 + 81$$

This can be done by adding term by term and you get.

$$1 + 3 + 9 + 27 + 81 = 121$$

Solution

Using the formula.

$$S_n = \frac{G_1(r^n - 1)}{r - 1}$$

from the series $G_1 = 1$

$r = 3$ $\quad |r| > 1$

$n = 5$

$$S_n = \frac{G_1(r^n - 1)}{r - 1} = \frac{1(3^5 - 1)}{3 - 1} = \frac{243 - 1}{2} = 121$$

The same as when we added term by term:

When $|r|$ is less than one our formula must be changed for easy calculation.

$$S_n = \frac{G_1(1 - r^n)}{1 - r} \qquad |r| < 1$$

Example

Find the sum of the first terms of the following GP.

$10 + 5 + 2.5 +$

Using the formula

$$Sn = \frac{G_1(1 - r^n)}{1 - r}$$

given $G_1 = 10$

$r = {}^1/_2$

$$S_5 = \frac{10\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \frac{1}{2}} = \frac{10(1 - \frac{1}{32})}{\frac{1}{2}} = 20(\frac{31}{32}) = 19.375$$

$S_5 = 19.375$

When $r = 1$ this means that there is no change of values for terms of the GP.

$G_1 = G_2 = G_3 = G_4 = \ldots. = G_n$

but

$S_n = G_1 + G_2 + G_3 + G_4 + G_5 + \ldots. + G_n$

Since all terms are equal to G_1

then $S_n = G_1 + G_1 + G_1 + G_1 + \ldots. + G_1$

$= nG_1$

$$S_n = nG_1 \qquad \text{for } r = 1$$

Where n is the number of terms.

Example

Find the sum of the first 10 term of the following GP.

$G_n = 2$

Solution

If $G_n = 2$

This means that $r = 1$

Therefore; $G_1 = G_2 = G_3 = G_4 = \ldots. = G_n = 2$

but $S_n = nG_1$

Therefore $S_{10} = 10 \times 2 = 20$

$\mathbf{S_{10} = 20}$

Example

Find the sum of the 8 terms of the GP with

$G_1 = 8 \qquad r = 2$

Solution

Using the formulae;

$$S_n = \frac{G_1(r^n - 1)}{r - 1}$$

$$S_8 = \frac{8(2^8 - 1)}{2 - 1} = 8(2^8 - 1)$$

$$= 8(256 - 1)$$

$$= 2040$$

$$\mathbf{S_8 = 2040}$$

SELF-CHECK EXERCISE 2

Find the sum of the ***n*** terms of the GP with the given values.

(1) $G_1 = 192, \quad A_4 = 3 \quad n = 4$

(2) $G_1 = 162 \quad r = -{}^1/_2 \quad n = 6$

(3) $G_1 = 8 \quad r = 2, \quad n = 5$

Compare your answers with those given at the end of the unit.

Summation of Geometric Progression and Infinite Series

Consider the geometrical progression whose common ratio is less than one, $|r| < 1$.

We said that the sum of the first n terms is given by $S_n = \frac{G_1(1 - r^n)}{1 - r}$

For expression $G_1 + G_2 + \ldots + G_{n-1} + G_n$ this is a finite series, so is known, but for the summation of $G_1 + G_2 + G_3 + \ldots + G_n + \ldots$ this is an infinite series and the summation has no limit; but when $|r| < 1$ the expression $S_n = \frac{G_1(1 - r^n)}{1 - r}$ becomes close to $\frac{G_1}{1 - r^{1}}$ for large value of n.

So we say the summation of $G_1 + G_2 + G_3 + \ldots + G_1 + \ldots$ becomes $\boxed{S_n = \frac{G_1}{1 - r}}$

This formula has many applications in finding expression as fractions in their lowest terms of number with repeating decimals.

Example

Write the following repeating decimal number into fraction:

$0.\dot{5}$ this means that 5 is repeated without ending.

Solution:

The number $0.\dot{5}$ can be written into geometric progression as $0.5 + 0.05 + 0.005 + 0.0005 + \ldots$

If you look carefully at the GP you can find r and G_1

$G_1 = 0.5$, $G_2 = 0.05$

$$r = \frac{G_2}{G_1} = \frac{0.05}{0.5} = \frac{5}{50} = \frac{1}{10}$$

you can use your formula for Sn.

$$S_n = \frac{G_1}{1 - r}$$

$$= \frac{0.5}{1 - \frac{1}{10}} = \frac{0.5}{\frac{9}{10}} = \frac{1}{2} \times \frac{10}{9} = \frac{5}{9}$$

Then $0.\dot{5} = \frac{5}{9}$ into fraction term.

$$\therefore \mathbf{0.\dot{5} = \frac{5}{9}}$$

Example

Express as fractions in their lowest terms

(ii) $0.0\dot{7}$

(iii) $0.\dot{4}\dot{5}$

Solution

(iii) 0.07 this can be written as $0.0777777...$ which may be written as

$$\frac{7}{100} + \frac{7}{1000} + \frac{7}{10000} + \frac{7}{100000} + \ldots$$

This is a Geometrical Progression with $G_1 = \frac{7}{100}$, $G_2 = \frac{7}{1000}$

$$r = \frac{G_2}{G_1} = \frac{7}{1000} \times \frac{100}{7} = \frac{1}{10}$$

$$S_n = \frac{G_1}{1 - r}.$$

$$= \frac{\frac{7}{100}}{1 - \frac{1}{10}} = \frac{7}{100} \times \frac{10}{9} = \frac{7}{90}$$

Therefore $0.0\dot{7}$ is equal to $\frac{7}{90}$

(ii) $0.\dot{4}\dot{5}$. this can be written as $0.45\ 45\ 45\ldots$ which may be written as

$\frac{45}{100} + \frac{45}{10000} + \frac{45}{1000000} + \ldots$ this is a geometric progression with

$$G_1 = \frac{45}{100} \text{ and } G_2 = \frac{45}{10000}$$

$$r = \frac{G_2}{G_1} = \frac{45}{10000} \times \frac{100}{45} = \frac{1}{100}$$

$$S_n = \frac{G_1}{1 - r} = \frac{\frac{45}{100}}{1 - \frac{1}{100}} = \frac{45}{100} \times \frac{100}{99} = \frac{5}{11}$$

Therefore $0.\dot{4}\dot{5} = \frac{5}{11}$.

SELF-CHECK EXERCISE 3

Express the following repeating decimal numbers as fractions

(1) $0.\dot{8}$

(2) $0.\dot{1}\dot{2}$

(3) $3.\dot{2}$

(4) $1.\dot{0}\dot{0}\dot{4}$

(5) $1.\dot{4}\dot{1}$

Compare your answers with those given at the end of the unit.

11.2.2 ARITHMETIC AND GEOMETRIC MEANS

Arithmetic Mean

You have learned about AP. For instance you have some data obtained from a number of experiments but during data recording you can accidentally omit some values which will not be easy to get back quickly. So we must develop the quickest way of getting the missing number using AP:

Consider the following sequence: 1, 3, 5, 7, ***a***, 11, 13.

Suppose ***a*** is a number whose value is not known, but by looking at the pattern you can guess that ***a*** is equal to **9**, or you can find the average of two numbers **7** and **11** to get ***a***.

that is $\quad a = \dfrac{7+11}{2} = \dfrac{18}{2} = 9$

Therefore 9 is the Arithmetic mean of 7 and 11

Generally if you have three consecutive numbers, ***a*, M, *b*** in the AP, the following relation is true.

$$M - a = d$$
$$b - M = d$$

Therefore $M - a = b - M$

Solving for M

$$M - a = b - M$$
$$M - a + M = b - M + M$$
$$2M - a = b$$
$$2M + a - a = b + a$$
$$2M = a + b \quad \text{dividing by 2 an each side.}$$
$$M = \frac{a+b}{2}$$

M is called the arithmetic mean of ***a*** and ***b***

Arithmetic Mean (M) of two numbers ***a*** and ***b***
$M = \dfrac{a+b}{2}$

<u>Example</u>:

Find the arithmetic mean of 10 and 12

$$M = \frac{a+b}{2} = \frac{10+12}{2} = 11$$

arithmetic mean is 11.

Geometric Mean

In GP sometimes terms are omitted in printing or otherwise, so we have to find the missing terms. Sometimes we use formula to calculate the nth terms, but sometimes it is very difficult to use that formula simply because ***n*** of that term is too big. Using formula takes a lot of time. Consider the following GP.

3 + 6 + 12 + 24 + 48 + 96 + 192 + 348 + 768 + $\underline{a}$ + 3072: ***a*** is missing may be by a printing error. If we use formulae to find that term, first we have to find the position the ***n*** of that term, then the use of the formula.

$G_n = G_1(r^{n1})$

Given G_1 = 3

r = 2

The missing term is 10^{th} term

$\therefore$ ***n*** **= 10**

$G_{10} = 3(2^{10\bar{1}1}) = 3\times2^9 = 1536$ to evaluate 2^9 is a very big task and the chances of making mistakes are also very big. So we develop another way of getting a solution using the two numbers close to the missing term.

Let ***a***, **M** and ***b*** be three consecutive terms of GP then the ratio of $\frac{M}{a} = \frac{b}{M}$ suppose **M** is the missing term.

Solving for M

$\frac{M}{a} = \frac{b}{M}$ by cross multiplication

$M^2 = ab$ Take square root of both numbers

$M = \sqrt{ab}$

The geometric mean of ***a*** and ***b*** is $M = \sqrt{ab}$

<u>Example</u>

Find the geometrical mean of 3 and 27

$M = \sqrt{ab}$ $a, = 3$ $b = 27$

$M = \sqrt{3 \times 27} = 9$

<u>Example</u>

Find the dimension of a square which is equal to the area of a rectangle of length 10m and width of 20m.

<u>Solution</u>

The dimension of the square using the idea of geometric mean:

$M = \sqrt{ab}$

$= \sqrt{10\text{m} \times 20\text{m}}$

= 14.14m

The dimension of the square is 14.14m × 14.14m

SELF-CHECK EXERCISE 4

Answer the following questions

1. Find the sum of the first ten terms of the GP: $2 - 4 + 8 - 32 + \ldots$

2. Find the sum of the first 5 terms of the following GP.
   (i) $2 + 4 + 8$
   (ii) $3 + 9 + 27$
   (iii) $5 + 25 + 125$

3. (i) Find the geometric mean of -6 and -54.
   (ii) A rectangle has the following dimensions 8m by 32m. Find the dimension of the square which will have the same area.
   (iii) Which figure in (ii) has the smallest perimeter?

4. Write each of the following repeating decimals as a fraction.
   (i) 0.14
   (ii) 1.414
   (iii) 0.3

5. Find the tenth term of a GP if the fourth term is 8 and the seventh term is 16.

6. What is the difference between the sums of the first 10 terms of the AP and GP whose first two terms are: $-2 + 4 \ldots$?

Compare your answers with those given at the end of the unit.

1. Geometric Progression (GP) is a series of numbers in which each number after the first can be obtained from the preceding one by multiplying it by a fixed number called the common ratio(r).

2. n^{th} term denoted by $\boldsymbol{G_n}$ is obtained by using the formulae: $G_n = G_1 r^{n} - 1$
   G_1 = first term
   r = common ration
   n = number of terms

3. Summation of terms denoted by $\boldsymbol{S_n}$ is obtained by

$$S_n = \frac{G_1(r^n - 1)}{r - 1} \quad \text{if } |r| > 1$$

$$S_n = \frac{G_1(1 - r^n)}{1 - r} \quad \text{if } |r| < 1$$

$$S_n = nG_1 \quad \text{if } r = 1$$

4. The geometric mean (M) of two numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ is $M = \sqrt{ab}$

11.2.3 COMPOUND INTEREST

You have already learnt about the method of calculating simple interest. But in business that cannot always be the case. Think of buying and selling things. For example, you can buy 10 pairs of khanga at 20,000sh. After selling them you get 25,000 shillings. This means your Principal value of 20,000 shillings generated a profit of 5,000/=. When you order another bale of khanga, you can use 25,000 shillings as your new Principal value and the profit after selling may be 6,000/=. You can see that at first you obtained a profit of 5,000, but in the second bale you obtained an extra 1,000/=. This means that the profit has generated more profit. This way of calculating profit or interest is known as Compound Interest. With our knowledge of GP we can develop a method of calculating our New Principal value after time T.

Using our example of method of The IAE Centre Coordinator of Kimanye .

Principal Value = 10,000sh.
Rate (R) = 10,
Time (T) = 5 years

Let us use the Simple Interest techniques to calculate the money gained annually.

Year 1, $I = \frac{PRT}{100} = \frac{10000 \times 10 \times 1}{100}$ = 1,000 shillings

At the end of year one the total amount of money in the bank will be

$P_1 = P + I_1$ = 10,000shs + 1,000sh. = 11,000shs

this will be your Principal at the beginning of year two.

Year 2: $I_2 = \frac{P_1RT}{100} = \frac{11000 \times 10 \times 1}{100}$ = 1,100shs

$P_2 = P_1 + I_2$ = 11,000 + 1,100 = 12,100shs.

Year 3: $I_3 = \frac{P_2RT}{100} = \frac{12100 \times 10 \times 1}{100}$ = 1,210shs

$P_3 = P_2 + I_3$ = 12,100 + 1,210 = 13,310shs

Year 4: $I_4 = \frac{P_3RT}{100} = \frac{13310 \times 10 \times 1}{100}$ = 1,331shs

$P_4 = P_3 + I_4$ = 13,310 + 1,331 = 14,641shs

Year 5: $I_5 = \frac{P_4RT}{100} = \frac{14641 \times 10 \times 1}{100}$ = 1,464.10shs

$P_5 = P_4 + I_5$ = 14,641 + 1,464.1 = 16,105.10shs

The money which our Head teacher will receive at the end of the fifth year will be 16,105.1shs. Comparing the two, Simple and Compound Interest.

Compound Interest	16,105.1sh
Simple Interest	− 15,000.0
	1,105.10shs

The compound interest is more beneficial by 1,105.10shs.

But this way of calculating interest and principal is very tedious, so let us develop a quick way of calculating money accumulated after a specific period by using the principal of Geometric Progression. Let us use our example again but this time without simplifying the obtained answer.

$P = 10{,}000$
$R = 10$
$I = \frac{PRT}{100} = 1{,}000$

$P_1 = P + I = 10{,}000 + \frac{10000 \times 10 \times 1}{100}$

we put brackets and take out the common number

$= 10{,}000(1 + \frac{10}{100})$
$= 10{,}000(1 + 0.1)$
$= 10{,}000(1.1)$

At the end of year two:

$P_2 = P_1 + I_1 = 10{,}000 \times (1.1) + \frac{10000 \times (1.1) \times 10 \times 1}{100}$
$= 10{,}000 \times (1.1) + 10{,}000(1.1) \times (0.1)$
$= 10{,}000 \times (1.1) \times (1 + 0.1)$
$= 10{,}000 \times (1.1)^2$

At the end of year three:

$P_3 = P_2 + I_2$
$= 10{,}000 \times (1.1)^2 + \frac{10000 \times (1.1)^2 \times 10 \times 1}{100}$
$= 10{,}000(1.1)^2(1 + \frac{10}{100})$
$= 10{,}000(1.1)^3$

The principal value at the end of year four:

$P_4 = P_3 + I_3$
$= 10{,}000 \times (1.1)^3 + \frac{10000 \times (1.1)^3 \times 10 \times 1}{100}$
$= 10{,}000(1.1)^3(1 + \frac{10}{100})$
$= 10{,}000(1.1)^4$

The amount at the end of year 5:

$P_5 = P_4 + I_4$
$= 10{,}000 \times (1.1)^4 + \frac{10000 \times (1.1)^4 \times 10 \times 1}{100}$
$= 10{,}000(1.1)^4(1 + \frac{10}{100})$
$= 10{,}000(1.1)^5$

This can be evaluated to get: 16,105.10 shillings

This is the same amount which we got when we calculated the Interest using our previous methods:

This can be summarised as.

$P_1 = 10,000(1.1)^1$
$P_2 = 10,000(1.1)^2$
$P_3 = 10,000(1.1)^3$
$P_4 = 10,000(1.1)^4$
$P_5 = 10,000(1.1)^5$

The principal value forms a Geometric Progression with $G_0 = 10,000$ and $r = (1.1)$.

In general, if P is the principal, R is the rate of interest, I is the interest, A is the amount after receiving the interest and T is the interest period then $A = P =$ Principal Value.

$$A_1 = P + I = \left(P + \frac{PR1}{100}\right)$$

$$= P\left(1 + \frac{R1}{100}\right)$$ This is the amount saved at the end of the first year saving

This amount will be treated as the new Principal value. The amount at the end of the second year will be A_2.

$$A_2 = P_1 + I_1 \text{ but } A_1 = P\left(1 + \frac{R1}{100}\right)$$

$$I_1 = P\left(1 + \frac{R}{100}\right)R$$

Therefore:

$$A_2 = P\left(1 + \frac{R}{100}\right) + P\left(1 + \frac{R}{100}\right)\frac{R}{100}$$ put brackets and take out the common terms.

$$= P\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right)$$

$$= P\left(1 + \frac{R}{100}\right)^2$$

$$A_3 = P_2 + I_2 \text{ but } A_2 = P\left(1 + \frac{R}{100}\right)^2$$

$$I = P\left(1 + \frac{R}{100}\right)\frac{R}{100}$$

$$= P\left(1 + \frac{R}{100}\right)^2 + P\left(1 + \frac{R}{100}\right)\frac{R}{100}$$

$$= P\left(1 + \frac{R}{100}\right)^2\left(1 + \frac{R}{100}\right)$$

$$= P\left(1 + \frac{R}{100}\right)^3$$

This can be generalised to

$$A_n = P\left(1 + \frac{R}{100}\right)^n$$

$$A_n = P\left(1 + \frac{R}{100}\right)^n$$

n = Time the money stayed in the bank
R = Rate
P = Principal
A_n = Amount of money after n years.

The profit after n year will be $I = A_n - P$
I = Profit (Interest)

Example
Find the compound interest of 20,000 shillings invested at the rate of 10% after five years.

Solution:
given P = 20,000 shs.
R = 10
n = 5

using the formulae $A_n = P\left(1 + \frac{R}{100}\right)^n$

$= 20{,}000\left(1 + \frac{10}{100}\right)^5$

$= 20{,}000(1.1)^5$

= 32,210.2 shillings

Interest: $A_n - P$ = (32,210.20 − 20,000.00) shillings
= 12,210.20 shillings

The Interest is 12,210.2 shillings after five years.

Example
Isanju Primary School invested a certain amount of money in a NMB Bank whose interest rate was 15% per year in a compound interest policy. After 4 years the bank record for the school went missing at the school but the amount of money in the bank was 55,000 shillings.

(i) How much money did the school deposit?
(ii) What was the Interest at the end of 4 years?

Solution
(i) Given A_4 = 55,000 shillings
n = 4 years
R = 15
P is missing

Using the formula

$$A_n = P\left(1 + \frac{R}{100}\right)^n$$

$$55{,}000 = P\left(1 + \frac{15}{100}\right)^4$$

$$= P(1 + 0.15)^4$$

$$= P(1.15)^4$$

$55{,}000 = 1.3225P$ divide by 1.3225 each side

$$P = \frac{55000}{1.3225} = 37{,}807.20\text{shs.}$$

This was the money invested

(ii) The Interest was $A_n - P$

$$= 55{,}000 - 37{,}807.20$$

$$= 17{,}192.80\text{shs.}$$

The Interest was 17,192.80shs.

SELF-CHECK EXERCISE 5

Answer the following questions

1. Find the Interest for a principal of 5,000 shillings at the rate of 15% per year in compound interest after 2 years.

2. Mary collected 6,000 shs as interest after investing at the rate of 12% compound annually for 4 years. What was her principal.

3. Mwanakwetu is a primary school teacher, She plans to buy 30 corrugated iron sheets at the rate of 5,000 shs per sheet in five years to come. If she decides to invest money in a bank which provides an interest rate of 10% compounded annually ,what amount of money must Mwanakwetu deposit in the bank to enable her to buy those iron sheets after five years.

4. Find the interest of 30,000 shs at the rate of 8% per annum simple interest after:
   (i) 4 years
   (ii) 6 years
   (iii) 10 years

Compare your answers with those given at the end of the unit.

11.2.5 POST TEST OF THE WHOLE UNIT

1. What is the 3rd term in the following sequence 5, 15____, 135

2. If 5, m, n, 40 are in GP find m and n

3. Given the following term, find the first term of GP and common ratio
   $G_6 = 16$
   $G_7 = 64$

4. What will a policeman who got a starting salary of 66,000 shillings and 10% increase each year for 8 years get after eight years of his service?

5. Given $G_1 = 8$ and $r = 2$. Find the sum of the 8 terms of the GP.

11.2.6 TUTOR MARKED ASSIGNMENT

Answer the following questions in the work book provided then send your answers to your Tutor for marking and commenting.

1. Find 40^{th} term of the G.P. 2,14, 18 **(10 Marks)**

2. Find the n^{th} term of the G.P. 27,81,243

   (10 Marks)

3. Find the geometric/mean of 18 and 72 **(10 Marks)**

4. If 5, x, y, 40 are in G.P Find x and y **(10 Marks)**

5. What is the n^{th} term of G.P 5, -5/, 5, -5/ ,…? 2,4,8… **(10 Marks)**

6. What is the 24^{th} term of the G.P 5, 15, 45…….. **(10 Marks)**

7. Find the nth term of the G.P 16, 8, 4, … … **(10 Marks)**

8. Find the 4^{th} term of the following series of G.P 81,27,9, …

   (10 Marks)

9. Find the sum of 5^{th} terms of the G.P. 2, -6, 18, -54…. **(10 Marks)**

10. What is the nth term of the G.P x, 2 x y, 4 x y^2, ….. **(10 Marks)**

11.2.7 KEY ANSWERS TO SELF-CHECK EXERCISES AND POST TEST

Self-Check Exercises

Exercise 1

1. (i) $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

   (ii) $\dot{2}4 + \dot{8}16 + 3\dot{2}64$

   (iii) $2 + \frac{1}{\dot{2}}\frac{1}{4} + \frac{1}{\dot{8}}\frac{1}{16}$

2. $r = 4 \quad G_1 = \frac{1}{64}$

3. 102,000 shillings

Exercise 2

1. $S_4 = 255$
2. $S_6 = 121.5$
3. $S_5 = 248$

Exercise 3

1. $\frac{8}{9}$
2. $\frac{4}{33}$
3. $\frac{29}{9}$
4. $\frac{10003}{999}$
5. $\frac{140}{99}$

Exercise 4

1. 682

2. (i) 62
   (ii) 363
   (iii) 3905

3. (i) 18
   (ii) 16m
   (iii) Square is having smallest perimeter

4. (i) $\frac{14}{99}$

   (ii) $\frac{157}{111}$

   (iii) $\frac{3}{10}$

5. 32

6. 432

<u>Exercise 5</u>

1. 1612.50 shillings
2. 10,000 shillings
3. At least 93,200 shillings
4. (i) 9,600 shillings
   (ii) 14,400 shillings
   (iii) 24,000 shillings

Post Test Checklist

1. 45
2. m = 10, n = 20
3. $r = 4 \quad g_1 = \frac{1}{64}$
4. 102,000 shillings
5. $S_8 = 2040$

UNIT 11.3

RATE AND VARIATION

CONTENTS

11.3.0.1 INTRODUCTION

In Module 10 you learned about the methods of connecting two elements (things) from one set to another set to form what you called RELATIONS.

In this Unit you will study the methods of comparing two quantities (or numbers) which are related to each other. The knowledge on RATIOS as it is discussed in the first section will be your starting point towards the direction of learning what are RATE and VARIATION.

11.3.0.2 OBJECTIVES

At the end of the unit you should be able to:

- relate quantities of different kinds;
- relate quantities of the same kind;
- solve problems on simple interest;
- solve problems on direct and inverse variations;
- use joint variation in solving problems.

11.3.1 RATE

You have already learned that **ratios** compare two quantities which are of the **same kind,** for example 4kg: 7kg or 3m: 10m. You will now learn that:

> RATE compares units of quantities of different kinds.

<u>Examples</u>

(i) A person is paid sh. 240 after working for 8 hours. In this example, the connection (comparison) of the amount the person is paid per working hour is said to be at the **rate** of sh. $\frac{240}{8}$ per hour or sh. 30 per hour.

(ii) A cyclist travels 18km in $1\frac{1}{2}$ hours. This is at the **rate** of $\frac{18}{1\frac{1}{2}}$ km an hour or 12 km an hour.

(iii) Two pencils cost ten shillings. You can say the price is at the **rate** of 2 pencils per 10sh. or 1 pencil per 5sh.

Please note that a rate may also connect quantities of the same kind, for example, if the price of a kilogram of sugar has been increased from sh.400 to sh.450, find the rate increase in cents per shilling.

<u>Solution</u>

Original Price = sh. 400
New Price = sh. 450
Increase in Price = sh. 450 – sh. 400 = sh. 50

$$\text{Rate increase} = \frac{50}{400} = \frac{50 \times 100\text{cts}}{400\text{sh.}}$$
$$= 12.5 \text{ cts per sh.}$$

∴ The rate increase in price is 12.5 cents per shilling. (Cents and shillings are quantities of the same kind).

SELF-CHECK EXERCISE 1

Do the following exercise:

1. Find the rate at which an egg is sold if a tray of 30 eggs costs 1500 shillings.

2. A radio repairer is paid 3000 shillings after servicing a radio for 8 hours. Find his rate per hour.

3. A knife originally marked sh. 87.50 is sold at sh. 66.50. Find the rate of decrease in price in cents per shilling.

4. A 1500-litre water tank takes $2\frac{1}{2}$ hours to fill. Find the rate of filling the tank in litres per minute

5. In a town with a population of 55000 there are 500 deaths in one year. Find the **death rate** per 1000 people in the town.

Compare your answers with those given at the end of the unit.

Simple Interest

When you deposit money in a Postal Bank or Co-operative Society Bank you are paid a profit for the use (by the bank) of your money. This profit money as a result of your deposited money is called an INTEREST (SIMPLE INTEREST). If you get, say, 10% on your deposited money, it means that you get sh. 10 for every sh. 100 you deposited for one year. The 10% is called the RATE of Interest (R). The rate is usually given annually (or once every year).

For example, the Interest on

- (i) sh. 1000 for 1 year at 10% per year is sh. 100
- (ii) sh. 3000 for 1 year at 10% per year is sh. 300
- (iii) sh. 3000 for 2 years at 10% per year is sh. 600

- INTEREST (I) is the amount of money paid as profit.
- PRINCIPAL (P) is the amount of money deposited.
- RATE (R%) is the percentage charge on the Principal per year.
- TIME (T) is the period in years over which the principal is deposited.

Formula:

$$INTEREST = \frac{PRINCIPAL \times TIME \times RATE}{100}$$

$$\mathbf{I = \frac{P \times T \times R}{100}}$$

Thus:

$$\text{PRINCIPAL, P} = \frac{100 \times I}{T \times R}$$

$$\text{TIME, T} = \frac{100 \times I}{P \times R}$$

$$\text{RATE, R} = \frac{100 \times I}{P \times T}$$

When you add the Interest to the Principal you get the total AMOUNT (A) payable to the depositor at the end of the year.

$$\text{AMOUNT} = \text{PRINCIPAL} + \text{INTEREST}$$

$$A = P + I$$

Example

Find the simple interest of sh. 1800 for 2 years at the rate of 6%

Solution

Time, T = 2 years
Rate, R% = 6%
Principal, P = sh. 1800

Now, Interest, $I = \frac{P \times T \times R}{100}$

$= \frac{\text{sh. } 1800 \times 2 \times 6}{100}$

$= \text{sh. } 18 \times 2 \times 6$

$= \text{sh. } 216$

∴ The simple Interest is sh. 216

Example

Find the principal that will earn sh. 9000 in 5 years at $4\frac{1}{2}\%$

Solution

$I = \text{sh. } 9000$

$R\% = 4\frac{1}{2}\%$

$T = 5\text{yrs}$

Since $I = \frac{PTR}{100}$

$P = \frac{I \times 100}{T \times R}$

$= \frac{\text{Sh. } 9000 \times 100}{5 \times 4\frac{1}{2}}$

$= \frac{\text{Sh. } 9000 \times 2 \times 100}{5 \times 9}$

$= \text{sh. } 1000 \times 2 \times 20$

$= \text{sh. } 40,000$

∴ The Principal is sh. 40,000

Example

Find the time in which sh. 3000 will earn sh. 600 at the rate of 5% interest.

Solution:

$P = \text{sh. } 3000, I = \text{sh.} 600, R\% = 5\%$

$T = ?$

Since, $I = \frac{PTR}{100}$

then $T = \frac{I \times 100}{P \times R}$

$= \frac{\text{sh.} 600 \times 100}{\text{sh. } 3000 \times 5\text{yrs}}$

$= 4 \text{ yrs}$

∴ The time is 4 years

Example

The interest of 4 years on a Principal of 9000 is sh. 1260. Find the rate.

Solution

P = sh. 9000, T = 4yrs

I = sh. 1260 R = ?

Since $I = \frac{PTR}{100}$

then $R = \frac{I \times 100}{P \times T}$

$= \frac{1260 \times 100\%}{9000 \times 4}$

$= 0.035 \times 100\%$

$= 3.5\%$

∴ The rate is 3.5%

SELF-CHECK EXERCISE 2

Answer the following questions.

1. Find the simple interest on
   (i) sh.8,000 for 1 year at 2%
   (ii) sh.6,000 for 2 years at 5%
   (iii) sh.7,000 for 5 years at 3.5%
2. Find the simple interest on sh.5400 deposited for 18 months at a rate of 12% per year.
3. Find the rate on a principal of sh.6200 invested for a period of $4\frac{1}{2}$ years if the interest is sh.930.
4. Find the number of years in which the interest on
   (i) sh.2,000 at 4% is sh. 480
   (ii) sh.4,000 at 3% is sh. 360
   (iii) sh.9,000 at 6% is sh. 1080
5. For how long should sh.6720 be invested at the rate of 5% to get an interest of sh.560?

Compare your answers with those given at the end of the unit.

Money Systems of Exchange

Each country has its own type of money or currency. You know that the Tanzanian money (currency) is the SHILLING (Tsh.). During business transactions with other countries the currency (money) of one country can be converted (changed) into the currency of another country. For example the Tanzanian shilling can be converted to Kenyan shilling or to the American Dollar, etc. and vice versa.

The conversions (exchanges) can be made by using agreed ratios called the RATE OF EXCHANGE. These rates of exchange may change from time to time depending on the economy of the country at any given period of time of exchange (See APPENDIX).

For example, the Daily News newspaper is sold at 300 Tanzanian shillings in Tanzania and 30 Kenyan shillings in Kenya. The ratio of the Tanzanian shilling to the Kenyan shilling here is Tsh.300 to Ksh.30 or Tsh.10 to Ksh1.The exchange rate for the Tanzanian shilling to the Kenyan shilling is therefore, Tsh.10 per Ksh.1 or Tsh.100 per Ksh.10.

Example 5

If the rate of exchange of the Tanzanian shilling is Tsh. 850 = 1.5 $ (USA Dollar) find, to the nearest dollar, how much you would get in exchanging for Tsh.48000.

Solution

Suppose you would get x\$

Then by equal ratios (proportion)

$$\frac{x\$}{\text{Tsh}48000} = \frac{1.5\$}{\text{Tsh}850}$$

or $$\frac{x}{48000} = \frac{1.5}{850}$$

$$x = \frac{1.5 \times 48000}{850}$$

$$= \frac{15 \times 480}{85}$$

$$= \frac{3 \times 480}{17}$$

$$= 84\tfrac{12}{17} = 84.706\$$$

∴ You would get 85\$. (To the nearest dollar)

Example

The currency (money) of Germany is the Deutsche Mark (DM). Suppose that the exchange rate is Tsh.300 = 2.5 DM. Find how much money you would get in exchange for Tsh.6,250,000.

Solution

The rate of exchange is 2.5 DM per Tsh.300.
Suppose you get x DM for Tsh.6,250,000.
In proportion:

$$\frac{x\text{DM}}{\text{Tsh.}6250000} = \frac{2.5\text{DM}}{\text{Tsh.}300}$$

$$x\text{DM} = \frac{\text{Tsh.}6250000 \times 2.5\text{DM}}{\text{Tsh. }300}$$

$$= \frac{6250000 \times 2.5}{300}\text{DM}$$

$$= 52083.3 \text{ DM}$$

∴ **You would get 52,083 DM (to the nearest DM).**

SELF-CHECK EXERCISE 3

Suppose the exchange rates for Tsh.100 are:

- 61.90 Kroner (Denmark currency)
- 600 pesetas (Spain)
- 10.5$ (dollars) (USA)
- 5.0f (pounds) (Britain)
- 26.80DM (West Germany)

Answer the following questions:

Make the following conversions, giving your answers correct to two decimal places.

(i) Tsh.120 into DM

(ii) Tsh 85.30 into pesetas

(iii) Tsh.100 into f

(iv) f950 into Tsh.

(v) 850 Kroner into Tsh.

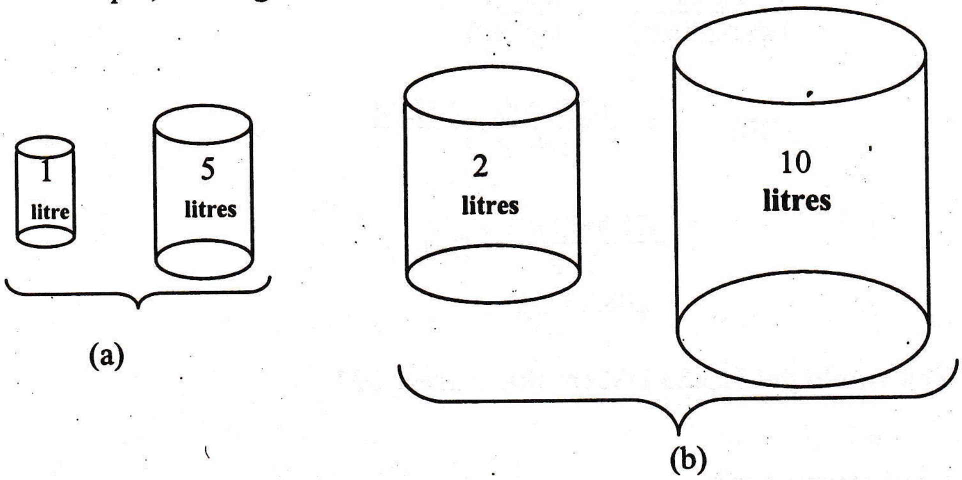
(vi) 8000$ into Tsh.

Compare your answers with those at the end of the unit.

11.3.2 PROPORTIONS

A PROPORTION is the equality of two ratios that is a statement that says that two ratios are equal.

For example, the diagrams below show (illustrate) equal ratios.



In diagram (a) 1 litre is $\frac{1}{5}$ of 5 litres and

in diagram (b) 2 litres is $\frac{2}{10}$ of 10 litres

The containers in (a) are said to be **proportional** to those in (b) since the ratio of their volumes are equal,

$\frac{1}{5} = \frac{2}{10}$ or the ratio 1:5 = 2:10

Many times we know three terms of a proportion and need to find the fourth term.

<u>Example</u>

Find the value of t in the following proportions.

(i) $\frac{5}{8} = \frac{t}{24}$ (ii) $\frac{3}{4} = \frac{12}{t}$

<u>Solution</u>

(i) $\frac{5}{8} = \frac{t}{24}$

$5 \times 24 = t \times 8$ (do cross product)

$8t = 120$

$t = \frac{120}{8}$

$= 15$

(ii) $\frac{3}{4} = \frac{12}{t}$

$3t = 12 \times 4$

$t = \frac{48}{3}$

$= 16$

or by using ratios of corresponding terms

(i) $5:8 = t:24$

$= t:3(8)$

$\therefore t = 3(5)$

$= 15$

(ii) $3:4 = 12:t$

$= 3(4):t$

$\therefore t = 4(4)$

$= 16$

Example

The recipe for a cake is 3 tablespoons of sugar to 2 cups of flour. How many tablespoons of sugar are required for 5 cups of flour?

Solution:

You let t be the number of tablespoons of sugar required for 5 cups of flour.

In ratios, 3 tablespoons:2cups = t tablespoons: 5cups

or $3:2 = t:5$

$$\frac{3}{2} = \frac{t}{5}$$

$$2t = 3(5)$$

$$t = \frac{15}{2}$$

$$= 7\frac{1}{2}$$

$\therefore 7\frac{1}{2}$ tablespoons of sugar are required for 5 cups of flour.

Example

If 6 razor blades cost sh. 180, what would be the proportional cost of 9 razor blades?

Solution:

Let s shillings be the proportional cost

Then

6 blades:sh180 = 9 blades: sh. s

$$\frac{6}{180} = \frac{9}{s}$$

$$6s = 9(180)$$

$$s = \frac{9 \times 180}{6}$$

$$= 270$$

$\therefore$ 9 razor blades cost sh.270.

SELF-CHECK EXERCISE 4

Answer the following questions:

1. Find the value of x in each of the following proportions.
   (i) $3:8 = 12:x$ (ii) $x:7 = 35:49$
   (iii) $x:6 = 5:8$ (iv) $7:9 = x:18$ (v) $5:13 = 35:x$

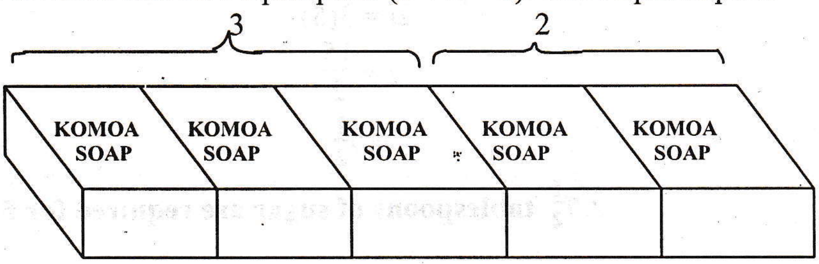
2. What number which when compared to 75 is the same as 6 compared to 25?
3. A piece of road 3cm long represents a distance of 18km on a map. Using the same scale, what is the length of a road that measures 2cm on the map?

4. If for every three men in a mathematics class there are two women, how many men are there in the class if there are eight women?

Compare your answers with those given at the end of the unit.

Proportional Parts

To divide a quantity into two parts which are in the ratio of 3:2, for example a bar of soap you first divide it into five equal parts ($3 + 2 = 5$). The required parts



will then be respectively 3 and 2 of the 5 equal parts, that is, the parts will be $\frac{3}{5}$ and $\frac{2}{5}$ of the whole bar.

Similarly if three friends share a pineapple in the ratio 2:5:7 then the proportions of the pineapple are $\frac{2}{14}$, $\frac{5}{14}$ and $\frac{7}{14}$, ($2 + 5 + 7 = 14$) of the whole pineapple.

Example
Divide 120 in a ratio 3:7.

Solution

$3+7 = 10$

The required proportional parts are;

$$\frac{3}{10} \text{ of } 120 = \frac{3}{10} \times 120 = 36$$

$$\text{and } \frac{7}{10} \text{ of } 120 = \frac{7}{10} \times 120 = 84$$

∴ The required parts are 36 and 84.

Example

Divide 156 in a ratio 3:4:5

Solution

3+4+5 = 12

The required proportional parts are

$\frac{3}{12} \times 156 = 3 \times 13 = 39$

$\frac{4}{12} \times 156 = 4 \times 13 = 52$

$\frac{5}{12} \times 156 = 5 \times 13 = 65$

∴ Proportional parts are 39, 52 and 65.

Example

Divide 299 into 3 parts in the ratio $\frac{1}{3} : \frac{5}{6} : \frac{3}{4}$

Solution

Ratio $\frac{1}{3} : \frac{5}{6} : \frac{3}{4}$ = 4:10:9 *(multiply by 12 the LCM of 3,6 and 4)*

Now 4+10+9 = 23

∴ Proportional parts;

$\frac{4}{23} \times 299 = 4 \times 13 = 52$

$\frac{10}{23} \times 299 = 10 \times 13 = 130$

$\frac{9}{23} \times 299 = 9 \times 13 = \frac{117}{299}$

SELF-CHECK EXERCISE 5

Answer the following questions:

1. Divide (i) 100 in the ratio 7:3
   (ii) 16 in the ratio 3:3:2
   (iii) $\frac{1}{2}$ in the ratio 4:1
2. Divide 600 shillings among Aisha, Juma , and John in the ratio 5:3:2 .
3. Four families share 28.6 kilograms of meat in the ratio 4:5:6:7. How many kilograms does each family get?
4. The volume of two tanks are in a ratio of 2:5. If the volume of the first tank is 2600 litres, find the volume of the two tanks together.

Compare your answers with those given at the end of the unit.

Direct Proportions

Suppose you let two quantities, say, P and Q be related to each other. If an increase; (or decrease) in amount of P causes the amount of Q to increase (or decrease) respectively in such a way that the ratio P:Q remains the same, then this relationship is called a DIRECT PROPORTION.

For example, if two tractors of the same kind can cultivate 40 hectares of land per day, then six tractors of the type can cultivate 120 hectares.

Note: $\frac{6 \text{ tractors}}{2 \text{ tractors}} = 3$ and $\frac{120 \text{ hectares}}{40 \text{ hectares}} = 3$

$\therefore \frac{6}{2} = \frac{120}{40}$ which shows you that this is an example of a **direct proportion**

Example

The total mass of 8 pieces of iron rods is 150kg. What will be the mass of 120 identical rods?

Solution

Rods to mass ratio is

8 rods : 150kg

or 1 rod : $\frac{150}{8}$ kg

$\therefore$ 120 rods have a mass of $\frac{150}{8} \times 120$kg

$= 2250$kg

Indirect Proportions

If the quantities of **P** and **Q** are related in such a way that **Q** decreases proportionally with an increase in **P**, then the relationship is called an **Inverse (or Indirect) proportion.**

For example, 100 pupils can eat a given amount of food in 30 days, but 200 pupils would eat the same amount of food in 15 days.

Note: the ratios $\frac{200}{100} = 2$ and $\frac{30}{15} = 2$

$$\therefore \frac{200}{100} = \frac{30}{15}$$

That is, the ratio of the **increase** of students is equal to the ratio of the **decrease** of the number of days in which the food is taken.

Example

Six tractors take 3 days to cultivate a 360-hectare farm. How long will nine tractors of the same kind take to cultivate the same farm?

Solution: 6 tractors: 3 days

1 tractor: 6×3 days (more days)

6×3

$\therefore$ 9 tractors take $\frac{6 \times 3}{9} = 2$ days

SELF-CHECK EXERCISE 6

Answer the following questions:

Calculate the unknown number x in each of the following proportions.

(i) $\frac{x}{4} = \frac{7}{12}$ (ii) $\frac{5}{x} = \frac{3}{4}$

(iii) $\frac{2}{5} = \frac{x}{7}$ (iv) $\frac{5}{12} = \frac{8}{x}$

2. A ship can sail 400 kilometres in 30 hours. How far can it go in 75 hours at the same speed?

3. A coil of wire has a mass of 300g. If a wire of the same material of length 25cm has 1.5 kg, calculate the length of the wire in the coil.

4. A man takes 15 days to dig a 6-hectare field. How long would 10 boys take to dig an 81-hectare field if 2 boys do the same amount of work as one man?

Compare your answers with those given at the end of the unit.

11.3.3 VARIATION

The words **"is proportional to"** are sometimes replaced by the words **"varies as"** when we are relating two quantities. For example, the statement **"*p* is proportional to *q*"** can be replaced by **"*p* varies as *q*"**

Direct Variation

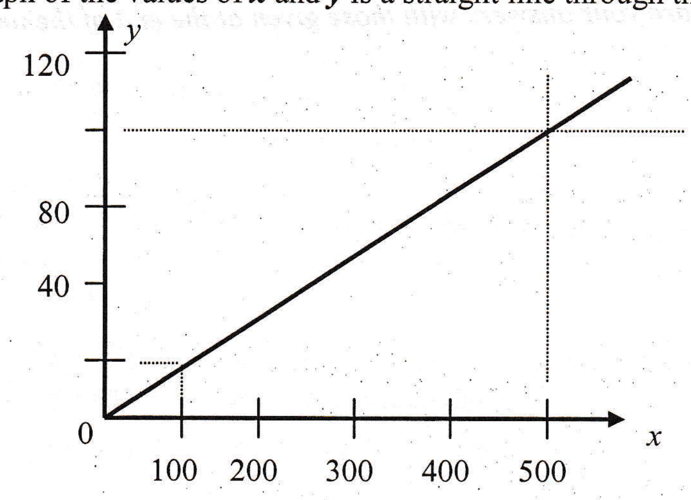
If *p* increases as *q* increases, we say that **"*p* is directly proportional to *q*"** or **"*p* varies directly as *q*"**

For example, the table below shows values of ***x*** and ***y*** recorded at different times. Study the table.

x	100	50	150	500	30
y	20	10	30	100	6

You observe that in each case the ratio ***x*:*y*** is the same, $\frac{x}{y} = 5$
or $x = 5y$.

This equation shows you that **"*x* varies directly as *y*"**.
Note the graph of the values of ***x*** and ***y*** is a straight line through the origin.



The statement **"*x* is directly proportional to *y*"** or **"*x* varies directly as *y*"** can be written in short as:

x, $\propto$ and y, where $\propto$ is a symbol for proportionality or variation. In the form of an equation you can write

$$\frac{x}{y} = k$$

or $x = ky$ where k is a constant of proportionality or variation

> If $x \propto y$, then $x = ky$ where k is a constant

Example

If $y \propto x$ and $y = 5$ when $x = 5$ find the value of y when $x = 3$ and the value of x when $y = 2$.

Solution

If $y \propto x$, then $y = kx$, where k is a constant

$y = 3$, when $= 5$, $\therefore\ 3 = k \times 5$

$$k = \frac{3}{5}$$

The equation is then $y = \frac{3}{5}x$

If $x = 3$, then $y = \frac{3}{5} \times 3$

$$= \frac{9}{5} = 1\frac{4}{5}$$

If $y = 2$, then $2 = \frac{3}{5}x$

$$x = \frac{2 \times 5}{3}$$

$$= \frac{10}{5} = 3\frac{1}{3}$$

Inverse Variation

If x decreases as y increases, we say that **"x is inversely proportional to y"** or **"x varies inversely as y"** and it is written in short as;

$$x \propto \frac{1}{y}$$

or $\quad x = \frac{k}{y}$, in equation from

or $\quad xy = k$, where k is a constant

For an inverse variation,

If $x \propto \frac{1}{y}$, then $xy = k$, where k is a constant

Example

x varies inversely as w. Write down an equation representing this.

Solution

$$x \propto \frac{1}{w}$$

$$\therefore \quad x = \frac{k}{w}$$

or $\quad wx = k$, where k is a constant.

Example

If y is inversely proportional to x, and $y = 10$ when $x = 7$; find y when $x = 3.5$

Solution

$$y \propto \frac{1}{x}$$

or $$y = \frac{k}{x}$$

$$k = xy$$

Now $y = 10$, $x = 7$, then $k = 10 \times 7$
$= 70$

and $x = 3.5$, $k = 70$

$$\therefore \quad y = \frac{70}{3.5} = 20$$

SELF-CHECK EXERCISE 7

Answer the following questions

1. Write each of the following as an equation
   (i) f varies directly as g
   (ii) m varies inversely as f
   (iii) $s \propto \frac{l}{t}$
   (iv) $t \propto f$

2. If x varies directly as y, complete the following table.

x	5	10	15	20	-
y	-	36	-	-	9

3. If m varies inversely as t, complete the following table.

m	5	10	15	20	-
t	-	30	-	-	10

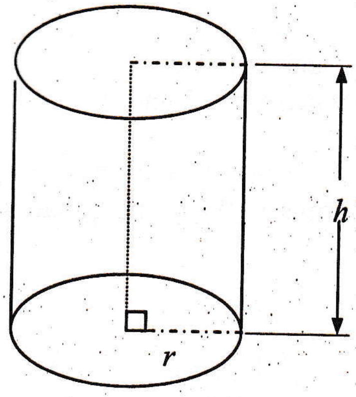
5. You are given that x is both directly proportional to y and inversely proportional to the square of z, complete the following table.

x	y	z
3	1	2
2		
1	3	

Compare your answers with those given at the end of the unit.

Joint Variation

You know that there are some equations which depend on more than one variable. For example, the equation of the volume of a cylinder of base radius ***r*** and height ***h*** is



$\mathbf{V} = \pi r^2 h$ *has two variables* **r** *and **h** where the π is connecting them to the volume* **V**

Cylinder

In many cases one variable depends upon the product of more than one variable. This is called <u>JOINT VARIATION.</u>

For example in the case of a cylinder "the volume **(V)** of the cylinder varies directly as the square of the base radius **(r^2)** and the height **(h)** of the cylinder", that is;

$$V \propto r^2 h$$

Thus; $V = kr^2h$

But $k = \pi$ in this case is a constant

∴ The equation of the Volume of the cylinder is

$$\boxed{V = \pi r^2 h}$$

<u>Example</u>

The resistance **R** of a wire varies directly to its length **L** cm and inversely to its cross section area ***a*** mm². When **L** is 2m and ***a*** is 4mm², the resistance **R** is 8750ohms. Calculate the value of **R** when **L** = 18m and ***a*** = 6mm².

<u>Solution</u>: Since $R \propto \frac{L}{a}$ (joint variation)

Then $\quad R = k\frac{L}{a}$, where k is a constant

$$\text{So } 8750 = \frac{k \times 2\text{m}}{4\text{mm}^2}$$

$$= \frac{k \times 2000\text{mm}}{4\text{mm}^2} \quad (1m = 1000mm)$$

$$k = \frac{4 \times 8750}{2000}$$

$$= \mathbf{17.5}$$

Now given, $L = 18\text{m}$, $a = 6\text{mm}^2$

$$R = \frac{17.5 \times 18 \times 1000}{6} \text{ohms}$$

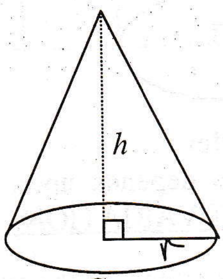
$= 52500\text{ohms}$ or $= 5.25 \times 10^4$ ohms

SELF-CHECK EXERCISE 8

Answer the following questions:

1. If w varies jointly as x and y and if $w = 48$ when $x = 3$ and $y = 4$, calculate the value of w when $x = 5.2$ and $y = 7.6$

2. The volume V of a right circular cone varies jointly as its height h and the square of the base radius r. Write the equation connecting V with r and h.



Cone

3. In working for 10 hours a day, 12 men can do a certain piece of work in 3 weeks. For how many hours a day must 15 men work in order to do the same amount of work in 14 days?

Compare your answers with those given at the end of the unit.

11.3.4 SUMMARY OF THE WHOLE UNIT

In this unit you have learnt some of the following important concepts

1. **RATIO** compares (by division) quantities of the same kind.
   Ratio can be expressed as a fraction $\frac{a}{b}$

2. **PERCENT** is a ratio with the second term 100.

3. **RATE** compares quantities of different kinds.

4. **SIMPLE INTEREST**

$$I = \frac{P \times T \times R}{100}$$

   Where P = Principal
   T = Time (years)
   R = Rate
   I = Interest

   AMOUNT = PRINCIPAL + INTEREST or A = P + I

5. **A PROPORTION** is a statement that two ratios are equal.
   The statement "x is proportional to y" is written in short as $x \propto y$

6. **DIRECT VARIATION:** If $x \propto y$, then $\frac{x}{y} = k$;
   where k is a constant of variation.

7. **INVERSE VARIATION:** If $x \propto \frac{1}{y}$, then $xy = k$;
   where k is a constant of variation.

11.3.5 POST TEST OF THE WHOLE UNIT

You are through with this unit, now do the following exercise which covers the whole unit:

1. Write the following ratios as fractions:

   (i) $3:4\frac{1}{2}$

   (ii) 0.3:0.9

   (iii) $1\frac{1}{2}:2\frac{1}{2}$

2. Complete the following ratios:

   (i) 3:12 = 9:__

   (ii) 0.5:4 = __:20

3. Find which ratio is greater:

   (i) 3:4 or 5:7

   (ii) 2:5 or 3:7

4. Write each of the following ratios as percentages:

   (i) $\frac{4}{5}$

   (ii) 3:10

   (iii) 0.25:1

5. A bicycle is bought for 50,000sh. and sold at 55,000sh.

   Find: (i) profit made

   (ii) the percentage of the profit

6. Find the buying price of a cow which was sold at 80,000 at 5% profit.

7. Juma invested 50,000sh. in a bank for five years at a rate of 10% simple interest. What amount of money did he collect at the end of the five years?

8. In June, 1998 the exchange rate for Tsh was 650/=sh per 1US$. How many US$ did Asha get for her trip to South Africa if she was given 3,000,000Tsh?

9. Divide 40 into the ratio of 2:3:2:1

10. The volume of a sphere varies directly as the cubic of its radius.

    (i) write the formula for finding the volume of the sphere

    (ii) if the constant, $k = \frac{4\pi}{3}$, find the volume of the sphere whose radius is 3cm.

Compare your answers with those given at the end of the unit.

11.3.6 TUTOR MARKED ASSIGNMENT

Now answer the following questions in the work book provided. Show all your work clearly and step by step. Remember to write your NAME, STUDENT NUMBER, ADDRESS, and UNIT NUMBER in front of your WORKBOOK, then send your workbook to your Tutor for marking and commenting.

1. Express the following ratios in lowest terms.
   (i) 144:81
   (ii) 160:96
   (iii) $14:\frac{1}{2}$
   (iv) $\frac{1}{4}:\frac{1}{7}$
   (v) 0.17:1 ***(5 Marks)***

2. Express the following ratios as percentages.
   (i) 1:200
   (ii) 2:5
   (iii) 49:500
   (iv) $\frac{3}{8}:\frac{1}{2}$
   (v) 0.75:5 ***(5 Marks)***

3. Express the following percentages as ratios in their lowest terms.
   (i) 72%
   (ii) $27\frac{1}{2}\%$
   (iii) $8\frac{1}{3}\%$
   (iv) 0.25%
   (v) 112% ***(5 Marks)***

4. A machine costing sh.18,000 is sold at a profit of 40%. What is the selling price? ***(5 Marks))***

5. Find the simple interest of sh.54000 invested for 18 months at the rate of 12% per year. ***(6 Marks)***

6. A shop has a sale, and a shirt originally marked sh.1750 is sold at sh.1320. Find the rate of the decrease in price in cents per shilling. ***(6 Marks)***

7. A layers mash is made of feeds A and B in the ratio 5:4. If 72 kg of this mixture are required, how much of each feed should be used? ***(8 Marks)***

8. A car uses petrol at the rate of one litre for every 11 km. If the price of petrol is sh.440 per litre, find the cost of petrol for a journey of 891 km.
(5 Marks)

9. A distance of 23 km is represented on a map by 9.2cm. Find the scale in km per cm. Find also the RF of the map.
(8 Marks)

10. If $y^2 \propto \frac{1}{x^3}$, and $y = 5$ when $x = \frac{1}{2}$, find y when $x = 4^1\frac{1}{2}$ and x when $y = 40$.
(8 Marks)

11. Convert 1760 Francs (French currency) into Tanzania shillings if the exchange rate is Tsh 100 = 45 Francs.
(8 Marks)

12. Find the principal that will earn sh.7290 in 8 years at $2^1/_2$% pear year.
(6 Marks)

13. A bicycle takes 18 hours to make a journey at 15 km per hour. At what speed will it have to travel to make the return journey in 16 hours?
(5 Marks)

14. Use the following table to say whether $y \propto x$, and if so write down the equation connecting x and y.

x	2.1	4.9	6.3	9.1
y	0.9	2.1	2.7	3.9

(10 Marks)

15. If w varies jointly as x and inversely as the square of y, and if $w = \frac{2}{3}$ when $x = 2$ and $y = 3$, calculate the value of w when $x = 5$ and $y = \frac{1}{2}$.
(10 Marks)

11.3.7 KEY ANSWERS TO SELF-CHECK EXERCISES AND POST TEST

Self check Exercises

Exercise 1:

1. sh. 150 per egg
2. sh. 375 per hr.
3. cts 24 per shilling
4. 10 litres per minute
5. 9 deaths per 1000 people.

Exercise 2

1. (i) sh. 160
   (ii) sh. 600
   (iii) sh. 1225

2. sh. 972

3. 3.3%

4. (i) 6 years
   (ii) 3 years
   (iii) 2 years

5. 1 year, 8 months or 20months

Exercise 3:

(i) 3 2.16DM
(ii) 511.80 Pesetas
(iii) ƒ5.00
(iv) Tsh. 19,000
(v) Tsh. 1,337.18
(vi) Tsh. 76,190.48

Exercise 4

1. (i) 2
   (ii) 5
   (iii) $3\frac{1}{4}$ or 3.25
   (iv) 21
   (v) 91

2. 18

3. 3.12 km

4. 12 men

Exercise 5

1. (i) 70, 30
   (ii) 6, 6, 4
   (iii) $\frac{2}{5}, \frac{1}{10}$
2. Aisha sh. 300
   Juma sh. 180
   John sh. 120
3. 5.2kg, 6.5kg, 7.8kg, 9.1kg
4. 65,000 ltr.

Exercise 6

1. (i) $2\frac{1}{3}$ or $\frac{7}{3}$
   (ii) $6\frac{2}{3}$ or $\frac{20}{3}$
   (iii) $2\frac{4}{5}$ or $\frac{14}{5}$ or 2.8
   (iv) $19\frac{1}{5}$ or 19.2
2. 1000km
3. 5cm
4. $40\frac{1}{2}$

Exercise 7

1. (i) $f = kg$
   (ii) $mf = k$ or $m = \frac{k}{f}$
   (iii) $s = \frac{kl}{t}$ or $st = kl$
   (iv) $t = kf$, where k is a constant
2.

x	5	10	15	20	2.5
y	18	36	54	72	9

3.

m	5	10	15	20	30
t	60	30	20	15	10

4.

x	y	z
3	1	2
2	6	6
1	3	6

Exercise 8

1. $w = 158.08$
2. $\frac{V}{r^2h} = k$ or $V = kr^2h$, where k is a constant
3. 12 days

Post Test Checklist

1. (i) $\frac{2}{3}$

   (ii) $\frac{1}{3}$

   (iii) $\frac{3}{5}$

2. (i) 36

   (ii) 2.5

3. (i) 3:4

   (ii) 3:7

4. (i) 80%

   (ii) 30%

   (iii) 0.0025%

5. (i) 5,000sh.

   (ii) 10%

6. 76190.5sh.

7. 75,000sh.

8. 4,615.4 US$

9. 10:15:10:5

10. (i) $V = kr^3$

    (ii) $36\pi\ cm^3$

INSTITUTE OF ADULT EDUCATION

OPEN AND DISTANCE LEARNING

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