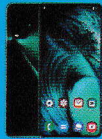
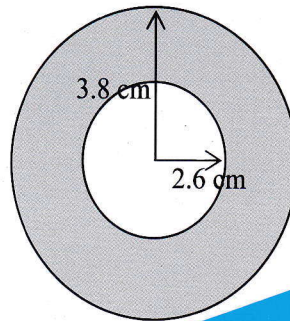
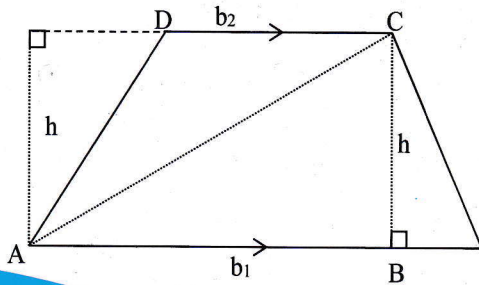


# BASIC MATHEMATICS

## Stage I

Proving Pythagoras Theorem, Trigonometric Ratios and Geometrical Transformations



Institute of Adult Education  
Alternative Secondary Education Pathway

# **BASIC MATHEMATICS**

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Ratios and Geometrical Transformations**

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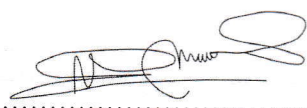
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**Institute of Adult Education**

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## About this module

This module has been produced by Institute of Adult Education. All modules produced by Institute of Adult Education are structured in the same way.

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## How this module is structured

### The module overview

The module overview gives you a general introduction to the module. Information contained in the module overview will help you determine:

- If the module is suitable for you;
- What you will already need to know;
- What you can expect from the module; and
- How much time you will need to invest to complete the module.

The overview also provides guidance on:

- Study skills;
- Where to get help;
- Module Unit assignments and assessments;
- Activity icons; and
- Units.

We strongly recommend that you read the overview *carefully* before starting your study.

### The module content

The module is broken down into units. Each unit comprises of:

- An introduction to the unit;
- Unit objectives;



- Unit outcomes;
- New terminologies;
- Core content of the unit with a variety of learning activities;
- A unit reflection;
- Unit assignments and/or assessments, as applicable; and
- Answers to Unit assignment and/or assessment.

## Resources

For those interested in learning more on this subject, we provide you with a list of additional resources at the end of this module. These may be books, articles or web sites.

## Your comments

After completing this module we would appreciate it if you would take a few moments to give us your feedback on any aspect of this module. Your feedback might include comments on:

- Module content and structure.
- Module reading materials and resources.
- Unit assignments.
- Module assessments.
- Module duration.
- Module support (assigned tutors, technical help, etc.)

Your constructive feedback will help us to improve and enhance this module.



## Module overview

---

### Welcome to this module

Dear learner, welcome to module 7 which has three units. Unit one deals with Verifying Pythagoras Theorem, the second unit discusses the Application of Trigonometric Ratios in solving real life problems and the third unit Demonstrates Geometrical Transformations.

I hope you will enjoy reading the module.

---

### General competence



Upon completion of this module you should be able to:

- Verify Pythagoras theorem,
- Apply trigonometric ratios and
- Demonstrate Geometrical transformations

---

### Study skills



As an adult learner your approach to learning will be different to that from your school days: you will choose what you want to study, you will have professional and/or personal motivation for doing so and you will most likely be fitting your study activities around other professional or domestic responsibilities.



Essentially you will be taking control of your learning environment. As a consequence, you will need to consider performance issues related to time management, goal setting, stress management, etc. Perhaps you will also need to reacquaint yourself to areas such as essay planning, coping with examinations and using the web as a learning resource.

Your most significant considerations will be *time* and *space* i.e. the time you dedicate to your learning and the environment in which you engage in that learning.

We recommend that you take time now—before starting your self-study—to familiarize yourself with these issues. There are a number of excellent resources on the web. A few suggested links are:

- <http://www.how-to-study.com/>

The “How to study” web site is dedicated to study skills resources. You will find links to study preparation (a list of nine essentials for a good study place), taking notes, strategies for reading text books, using reference sources, test anxiety.

- <http://www.ucc.vt.edu/stdysk/stdyhlp.html>

This is the web site of the Virginia Tech, Division of Student Affairs. You will find links to time scheduling (including a “where does time go?” link), a study skill checklist, basic concentration techniques, control of the study environment, note taking, how to read essays for analysis, memory skills (“remembering”).

- <http://www.howtostudy.org/resources.php>

Another “How to study” web site with useful links to time management, efficient reading, questioning/listening/observing skills, getting the most out of doing (“hands-on” learning), memory building, tips for staying motivated, developing a learning plan.

The above links are our suggestions to start with on your way. If you want to look for more go to [www.google.com](http://www.google.com) and type “self-study basics”, “self-study tips”, “self-study skills” or similar.



---

## Need help?



Dear learner, in the course of your study, you may need help in various issues such as the location and how to get support from resource centres, clarification of various issues pertaining to your study materials (modules) and so on. If this happens, you are advised to ask for the help from your centre coordinator or facilitator, you can also visit the website of the Institute of Adult Education which is [www.iae.ac.tz](http://www.iae.ac.tz) or ask for help by using phone no. +255 22 2150838.

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## Module assessment



After each unit, you will be required to attempt one unit assignment. These are not meant for submission rather for reflection on what you have learned in the whole module. You will also be given tests and assignments for submission as you will be guided by your module facilitator. You will also sit for mock examinations to accomplish your continuous assessment.

























## Getting around this module

### Margin icons

While working through this module you will notice the frequent use of margin icons. These icons serve to “signpost” a particular piece of text, a new task or change in activity; they have been included to help you to find your way around this module.

A complete icon set is shown below. We suggest that you familiarize yourself with the icons and their meaning before starting your study.

 Activity	 Assessment	 Unit assignment	 Case study
 Discussion	 Group activity	 Help	 Note it!
 Outcomes	 Reading	 Reflection	 Study skills
 Reflection	 Terminology	 Time	 Tip
 Computer-Based Learning	 Audio	 Video	 Feedback
 Objectives	 Basic Competence	 Answers to Assessments	



## Unit 1

---

### Verifying Pythagoras Theorem

#### Introduction

Dear learner, this unit derives and proves Pythagoras theorem. The knowledge of this section is very important in our lives. Do you know where this Pythagoras theorem can be applied? Think of construction of the buildings, bridges and roofing of houses and

reflect how this theorem can be applied. In this unit, you are therefore going to study about the definitions, derivations and the application of Pythagoras theorem and its application in solving various problems.

#### Learning Outcomes



Upon completion of this unit you should be able to:

- Identify angles in different figures;
- Measure angles using mathematical instruments;
- Construct lines and angles in different structures; and
- Solve various life related problems using Pythagoras theorem.



## Pythagoras Theorem

Pythagoras theorem is the theorem which was first derived by one of the great Mathematicians whose name was Pythagoras. The theorem states that: "the sum of squares of other two sides of the right-angled triangle is equal to the square of the Hypotenuse."

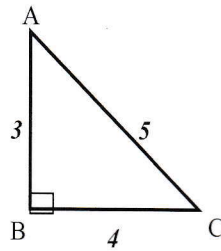


Figure 1.1 a

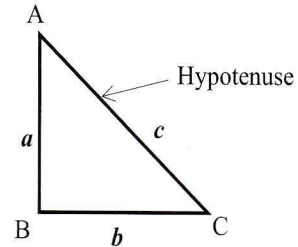


Figure 1.1 b

From Figure 1(a), we see that  $3^2 + 4^2 = 5^2$

This can be generalized from Figure 1 (b)

From Figure 1b above you will see that

$$AB^2 + BC^2 = AC^2$$

Since  $AB = a$  and  $BC = b$  and  $AC = c$

Then  $a^2 + b^2 = c^2$  this is the Pythagoras theorem

Pythagoras theorem is the theorem which states that *for any right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides*. This definition can be illustrated by Figure 1.2.

Illustration of Pythagoras theorem

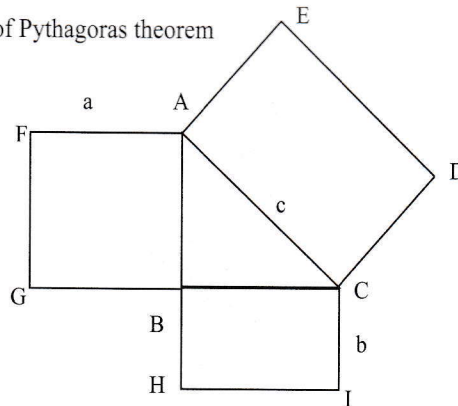


Figure 1.2



Consider the area of the squares

(i) Square ABGF which is on  $\overline{AB}$  : Area =  $a^2$

(ii) Square BCIH which is on  $\overline{BC}$  : Area =  $b^2$

(iii) Square ACDE which is on  $\overline{AC}$  : Area =  $c^2$

Therefore,  $a^2 + b^2 = c^2$

### Deriving Pythagoras Theorem

Draw a square and a trapezium, naming their sides. By considering their areas of the square and trapezium, derive Pythagoras theorem. Then compare your work with the examples given in Figure 3 and Figure 4. If you're able congratulation, if not, do more exercises and consult any competent person in mathematics. Now, go on reading by following each step in this part.

The Pythagoras theorem can be derived by using two methods.

These methods are:

(a) By using the Area of a square

(b) By using the Area of a trapezium

By using the Area of a Square on the given sides :

You can derive the Pythagoras theorem by using the area of the square in Figure 1(b). Consider the square as illustrated in below.

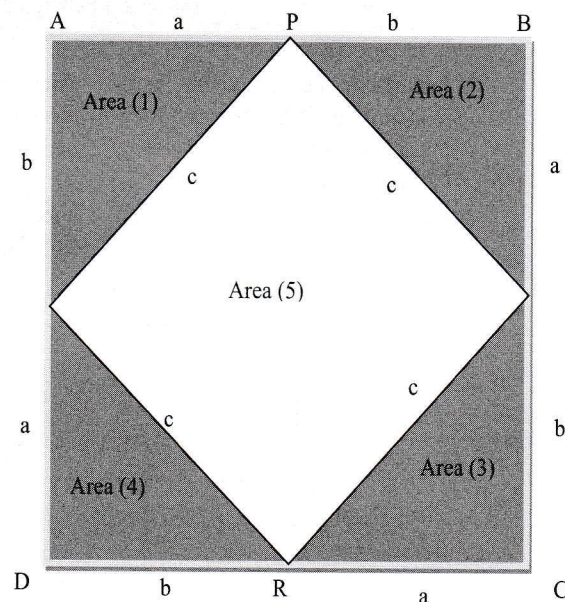




Figure 1.3

Consider the area of the whole square ABCD.

Where  $AB = a + b$ ,  $BC = a + b$ ,  $CD = a + b$  and  $DA = a + b$

$$\begin{aligned} \text{Area of ABCD} &= AB \times BC \\ &= (a + b)(a + b) \\ &= (a + b)^2 \\ &= a^2 + b^2 + 2ab \dots\dots\dots(i) \end{aligned}$$

Consider the 4 areas of the triangles

Area 1 =  $\frac{1}{2} ab$ , Area 2 =  $\frac{1}{2} ab$ , Area 3 =  $\frac{1}{2} ab$ , Area 4 =  $\frac{1}{2} ab$  and Area 5 =  $C^2$

$$\begin{aligned} \text{Total area of triangles} &= (1+2+3+4+5) = \frac{1}{2} ab + \frac{1}{2} ab + \frac{1}{2} ab + \frac{1}{2} ab + C^2 \\ &= (\frac{1}{2} ab \times 4) + C^2 \\ &= 2ab + C^2 \end{aligned}$$

$$= 2ab + C^2 \dots\dots\dots(ii)$$

But equation (i) is equal to equation (ii)

$$a^2 + b^2 + 2ab = c^2 + 2ab \dots\dots\dots \text{subtract } 2ab \text{ both sides}$$

$$a^2 + b^2 + 2ab - 2ab = c^2 + 2ab - 2ab$$

$$\therefore a^2 + b^2 = c^2$$

$a^2 + b^2 = c^2$

The Pythagoras theorem derived

**By using the Area of a Trapezium**

You can derive the Pythagoras theorem by using the area of a trapezium. Consider trapezium as given in Figure 1.4.

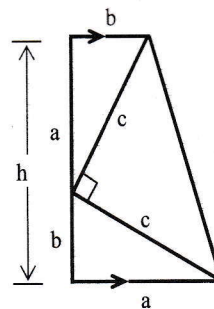


Figure 1.4



From Figure 4 above you will see that the area of a trapezium is given as:

$$\text{Area of Trapezium} = \frac{1}{2} h (a + b)$$

$$A = \frac{1}{2} h (a + b)$$

$$\text{But } h = a + b$$

$$\text{Area} = \frac{1}{2} (a + b) (a + b)$$

$$\text{Area} = \frac{1}{2} (a + b)^2.$$

By equating the area of Trapezium and the areas of triangles inside the trapezium you will have the following:-

$$\frac{1}{2} (a + b)^2 = \frac{1}{2} ab + \frac{1}{2} ab + \frac{1}{2} c^2$$

$$\frac{1}{2} (a^2 + 2ab + b^2) = \frac{1}{2} ab + \frac{1}{2} ab + \frac{1}{2} c^2$$

If you multiply both sides by 2 you will have.

$$2 \times \frac{1}{2} (a^2 + 2ab + b^2) = 2 (\frac{1}{2} ab + \frac{1}{2} ab + \frac{1}{2} c^2)$$

$$a^2 + 2ab + b^2 = ab + ab + c^2$$

$$a^2 + 2\cancel{ab} + b^2 = 2\cancel{ab} + c^2$$

$$\boxed{a^2 + b^2 = c^2}$$

The Pythagoras theorem derived

### Proof of Pythagoras Theorem

Dear learner, draw a right angled triangle, identify the angles present, draw the perpendicular line which will give you similar triangles. Can you calculate the length of the sides? How? Are the lengths the same? How? What about the angles? Are your answers the same with those given in the next part of your unit? Ok, make the follow up in the steps given in the solutions for better understanding.



### Calculations of length of sides of right angled triangle

The Pythagoras theorem can be proved by considering a Right-angled triangle ABC as given in Figure 1.5.

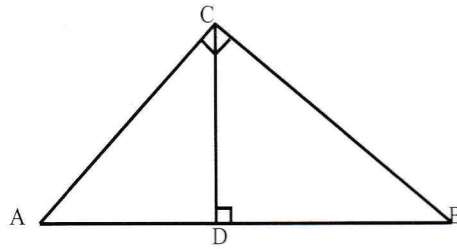


Figure 1.5

#### Solution

**Aim:** To prove that  $(AB)^2 = (AC)^2 + (BC)^2$ .

**Given:** ABC, angle  $ACB = 90^\circ$

**Construction:** Through C, draw CD such that CD is perpendicular to AB.

#### Proof:

(i) From Triangle ADC and ABC in Figure 5, we have:

$$m(\hat{ADC}) = m(\hat{BDC}) = 90^\circ \text{ (given and by construction)}$$

$$m(\hat{CAD}) = m(\hat{ABC}) \text{ (third angle of a triangle)}$$

$\therefore \Delta ABC \sim \Delta ACD$  (By AA Similarity theorem).

Since the triangles are similar then:

$$\frac{AB}{AC} = \frac{AC}{AD} = \frac{BC}{CD}$$

It follows that:

$$\frac{AB}{AC} = \frac{AC}{AD} \text{ or } \frac{AB}{AC} = \frac{BC}{CD}$$

$$\frac{AB}{AC} = \frac{AC}{AD} \Rightarrow AC \times AC = AB \times AD$$

$$(AC)^2 = AB \times AC \text{ - call it equation (i).}$$

(ii) Also in triangles ABC and CBD

$$m(\hat{ACB}) = m(\hat{CDB}) = 90^\circ$$



$m(\hat{CAB}) = m(\hat{CBD})$  third angles of a triangle.

$\therefore \Delta ABC \sim \Delta CBD$  (By AA -theorem) then since the triangles are similar

It follows that:

$$\frac{AB}{CB} = \frac{BC}{BD} = \frac{AC}{CD}$$

$$\frac{AB}{CB} = \frac{BC}{BD} \text{ or } \frac{AB}{CB} = \frac{AC}{CD}$$

Then,  $\frac{AB}{CB} = \frac{BC}{BD} \Rightarrow CB \times BC = AB \times BD$

$$(BC)^2 = AB \times BD. \text{ call it equation (ii).}$$

If you combine by adding (i) and (ii) above you will have the following:

$$\begin{array}{r} (AC)^2 = AB \times AD \\ + (BC)^2 = AB \times BD \\ \hline (AC)^2 + (BC)^2 = AB \times AD + AB \times BD \end{array}$$

Factor out AB in R.H.S since it is common to both sides gives:

$$(AC)^2 + (BC)^2 = AB(AD + BD)$$

but  $AD + BD = AB$

$$\therefore (AC)^2 + (BC)^2 = AB \times AB$$

$$(AC)^2 + (BC)^2 = (AB)^2$$

$$(AB)^2 = (AC)^2 + (BC)^2$$

Hence proved

The converse of the theorem is that, if the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the angle between those sides is a right angle, is also true.

## Applications of Pythagoras Theorem

Dear learner, do you know where the knowledge of the Pythagoras theorem is applied? If so give examples. Observe the arrangement of thatches of the newly constructed building or a roof of a house with no ceiling board. Identify different shapes, make their drawings and name them. Where else



can you observe similar Figures other than that in your surroundings? Give examples. I hope you can write structures designed by welders like gates, grills as well as other furniture. There are many corner angles formed in these structures. Read the following example thoroughly.

### Example

- In Figure 6, find AD if AC = 17cm, BC = 8cm and CD = 12cm.

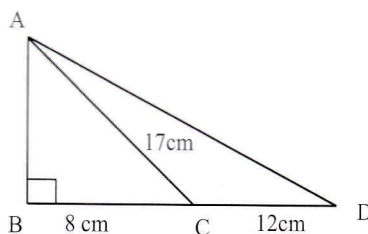


Figure 1.6

### Solution

If you apply the Pythagoras theorem to the  $\Delta ABC$

$$(AB)^2 + (BC)^2 = (AC)^2$$

$$(AB)^2 = (AC)^2 - (BC)^2$$

$$(AB)^2 = (17)^2 - 8^2$$

$$(AB)^2 = 289 - 64$$

Take square root of both sides

$$\sqrt{(AB)^2} = \sqrt{225}$$

$$AB = 15$$

$$\therefore AB = 15\text{cm}$$

Applying Pythagoras theorem again to  $\Delta ABD$  you will get.

$$(AB)^2 + (BD)^2 = (AD)^2$$

$$(AD)^2 = (AB)^2 + (BD)^2 \quad (\text{note that } BD = BC + CD = 20)$$

$$(AD)^2 = (15)^2 + (20)^2$$

$$(AD)^2 = 225 + 400$$

$$(AD) = \sqrt{625}$$

$$\therefore AD = 25\text{cm}$$



2. In Figure 7, prove that  $m(\hat{P}NR) = 90^\circ$  and hence calculate NR.

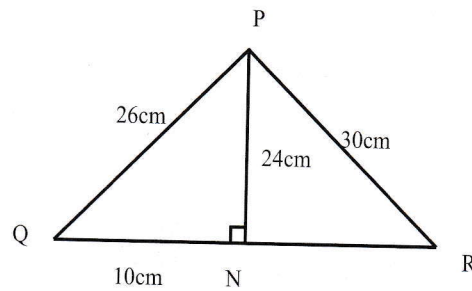


Figure 1.7

**Solution**

- (i) In  $\Delta QNP$  if you apply Pythagoras theorem you will get.

$$QN^2 + NP^2 = QP^2$$

$$10^2 + 24^2 = 26^2$$

$$100 + 576 = 676$$

$$676 = 676$$

Since  $(QN)^2 + (NP)^2 = (QP)^2$  then

$$m(\hat{P}NQ) = 90^\circ,$$

and hence  $m(\hat{P}NR) = 90^\circ$  because it is supplementary to  $m(\hat{P}NQ)$ .

- (ii) To find NR

By applying Pythagoras theorem.

$$(PN)^2 + (NR)^2 = (PR)^2$$

$$(24)^2 + (NR)^2 = 30^2$$

$$(NR)^2 = 900 - 576$$

$$NR = \sqrt{324}$$

$$\therefore NR = 18\text{cm.}$$

3. In Figure 8, ABC is a right angled triangle, A is a right angle and AD is an altitude.



- (a) Name two triangles which are similar to  $\triangle ABD$  and give reasons.  
 (b) Show that  $y^2 = uv$ ,  $x^2 = uw$  and  $xz = yw$ .

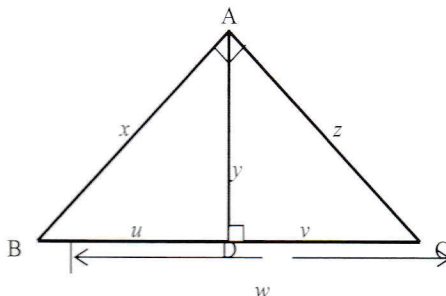


Figure 1.8

**Solution**

- (a)  $\triangle ABD$  is similar to:  $\triangle ACD$ ,  $\triangle CBA$  (by RHS)  
 (b) (i) To show that  $y^2 = uv$

In triangle  $ABD$  and  $\triangle CBA$ .

By applying Pythagoras theorem in  $\triangle ABD$  and  $\triangle ADC$

$$(AD)^2 + (BD)^2 = (AB)^2$$

$$y^2 + u^2 = x^2$$

$$y^2 = x^2 - u^2$$

$$y^2 + u^2 = x^2$$

$$(AD)^2 + (DC)^2 = (AC)^2$$

$$y^2 + v^2 = z^2$$

$$x^2 + z^2 = w^2$$

but  $w = u + v$

$$x^2 + z^2 = (u + v)^2$$

$$x^2 + z^2 = u^2 + 2uv + v^2$$

but  $z^2 = y^2 + v^2$

$$x^2 + (y^2 + v^2) = u^2 + 2uv + v^2$$

$$x^2 + y^2 + \cancel{v^2} = u^2 + 2uv + \cancel{v^2}$$

$$x^2 + y^2 = u^2 + 2uv$$

but  $x^2 = u^2 + y^2$

$$u^2 + y^2 + y^2 = u^2 + 2uv$$



$$u^2 + 2y^2 = u^2 + 2yv$$

$$\frac{2y^2}{2} = \frac{2uv}{2}$$

$$\therefore y^2 = uv \text{ Hence shown.}$$

(b)(ii) To show that  $x^2 = uw$ .

By applying Pythagoras theorem.

$$x^2 + z^2 = w^2 \text{ in } \Delta ABC$$

$$u^2 + y^2 = x^2 \text{ in } \Delta ABD$$

$$y^2 + v^2 = z^2 \text{ in } \Delta ADC$$

$$y^2 + v^2 = z^2$$

$$\text{but } v = w - u$$

$$y^2 + (w - u)^2 = z^2$$

$$y^2 + w^2 - 2uw + u^2 = z^2$$

$$\text{but } w^2 = x^2 + z^2 \text{ then}$$

$$y^2 + x^2 + z^2 - 2uw + u^2 = z^2$$

$$y^2 + x^2 - 2uw + u^2 = 0$$

$$y^2 + x^2 + u^2 = 2uw$$

$$\text{but } u^2 = x^2 - y^2 \text{ then}$$

$$y^2 + x^2 + x^2 - y^2 = 2uw$$

$$\frac{2x^2}{2} = \frac{2uw}{2}$$

$$\therefore x^2 = uw \text{ Hence shown.}$$

(b)(iii) To show that  $xz = yw$ .

$\Delta ADB \sim \Delta CAB$  by AA Similarity.

$$\text{Then } \frac{AD}{CA} = \frac{AB}{CB} = \frac{DB}{AB}$$

$$\frac{y}{z} = \frac{x}{w} = \frac{u}{x}$$



In particular this means that

$$\frac{y}{z} = \frac{x}{w} \quad \text{or} \quad \frac{y}{z} = \frac{u}{x}$$

then equate  $\frac{y}{z} = \frac{x}{w} \Rightarrow yw = zx$

$\therefore zx = yw$  Hence shown.

**Activity 1**

Using Pythagoras theorem, answer the following questions:

- Calculate the sides which are not given on the following right-angled triangles. Compare your steps in your work with the solutions shown in the examples above.

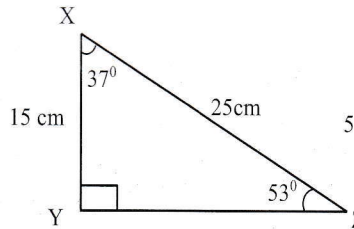


Figure 1.9 (a)

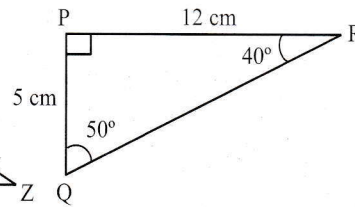


Figure 1.9 (b)

- 
- 

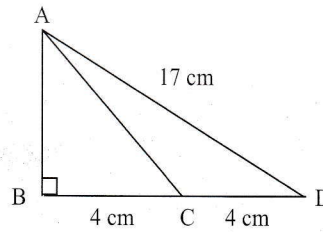


Figure 1.10

- Calculate (i) AB.  
(ii) AC

- Name two triangles similar to  $\Delta ABC$  in Figure 1.11

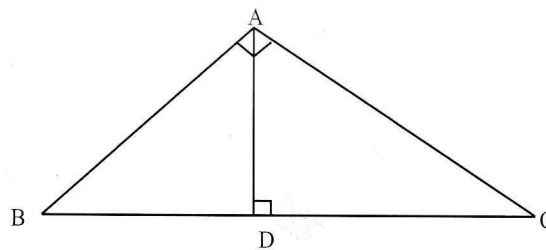


Figure 1.11

Compare your answers with those given at the end of the unit.



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## Unit Reflection



Dear learner, you are now through with this Unit, What are the most important concepts you have learnt? What are the challenging concepts that you need more time to learn better? What would you do to ensure that you clear the doubts on difficult areas? Can you use the knowledge from this unit in your daily life activities?

---

## Unit Assignment



Do the following questions:

- Given the triangle ABC, where  $B = 90^\circ$ . Find the length of the sides which are not given.
  - $AC = 26\text{cm}$      $AB = 10\text{cm}$
  - $AB = 20\text{cm}$      $BC = 21\text{cm}$
  - $AC = 25\text{cm}$      $AB = 15\text{cm}$
  - $AB = 28\text{cm}$      $BC = 45\text{cm}$
- The sides of a rectangle are 5cm and 7cm. Find the length of the diagonal.
- A ladder just reaches the top of a wall 18m high. If the end of the ground is 8m from the wall, find the length of the ladder.



4. By using the trapezium's area, prove that

(a)  $A^2 + b^2 = C^2$

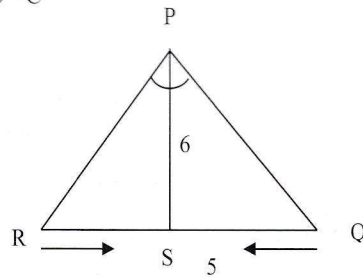


Figure 1.12

- (b) From Figure 12 calculate

- (i) PR      (ii) PQ

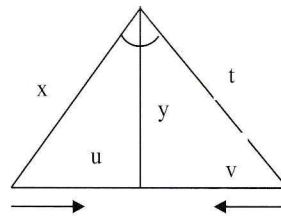


Figure 1.13

- (c) From Figure 13 show that:

- (i)  $y^2 = UV$   
(ii)  $x^2 = UP$

5. In triangle ABC  $A = 90^\circ$   $AB = 3.2\text{cm}$  and  $BC = 4.8\text{cm}$ , then find AC.
6. PQRS is a quadrilateral in which  $\hat{PQR}$  and  $\hat{PRS}$  are right angles. If  $PQ = 12\text{cm}$ ,  $RS = 9\text{cm}$ ,  $RS = 8\text{cm}$ . Find the length of PS and the area of PQRS.
7. The sides of a rectangle are 5cm and 5cm and 5cm. Find the length of a diagonal.

**Key Answers for Activity**

1. (a)  $YZ = 20$  (b)  $QR = 13$
2. (i)  $AB = 15$  (ii)  $AC = \sqrt{241}$
3. (i) BDA (ii) CDA



## Unit 2

### Applying Trigonometric Ratios

#### Introduction

Dear learner, now you are in unit two of Module 7. In this unit you will learn about sine, cosine and tangents of an angle. Trigonometric ratios are the geometry of sine, cosine and tangent of angles. The knowledge of this section is very important in our lives. Buildings, bridges and roofing of houses are made by the application of this knowledge. This unit deals with the definitions, derivations and the application of trigonometric ratios in solving various problems. I hope you will enjoy learning Mathematics from this unit.

#### Learning outcomes



Upon completion of this unit you should be able to:

- *Define* sine, cosine and tangent of an angle;
- *Find* sine, cosine and tangent of an angle without using tables;
- *Solve* trigonometric ratios related problems of special angles; and
- *Demonstrate* ability in constructing various structures involving trigonometry.

#### Definition

*Trigonometry* is a branch of mathematics that deals with the relations between the sides and angles of triangles. We have three common trigonometric ratios, i.e. Sine, Cosine and Tangent of angles.

Therefore, in order to understand clearly what is sine, cosine and tangent of angles, it is good if you consider the right-angled triangle ABC as shown in Figure 2.1.

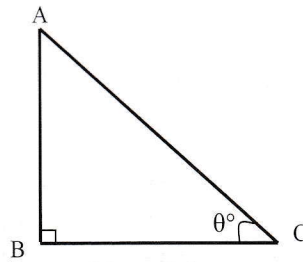


Figure 2.1

From Figure 14, I hope you know that side AC is always called the *Hypotenuse*. Side AB is the *opposite side* to Angle C and side BC is the *adjacent side* to angle C. Side BC is the *opposite side* to angle A and also side AB is the *adjacent side* to angle A. You can now see that the opposite side and adjacent side change according to the angle considered.

Hence, by referring to Figure 14 the sine, cosine and tangent of the angle  $\theta^\circ$  can be defined as follows:

## Trigonometric ratios

### Sine of angles

The sine of any angle is normally given by the ratio:

$$\text{Sine } \theta^\circ = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

Thus,  $\sin \theta^\circ = \frac{AB}{AC}$  hence in general the sine of any angle  $\theta^\circ$  is given by the following ratio:

$$\text{Sin } \theta^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

### Cosine of angles

The cosine of any angle is normally given by the ratio:

$$\text{Cosine } \theta^\circ = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$



Thus, from Figure 14, cosine  $\theta^\circ = \frac{BC}{AC}$

Hence, in general the cosine of any angle  $\theta^\circ$  is given in short by the ratio:

$$\cos \theta^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

### Tangent of angles

The tangent of any angle is normally given by the ratio:

$$\text{Tangent} \theta^\circ = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

Thus, from Figure 14 tangent  $\theta^\circ = \frac{AB}{BC}$

Hence in general the tangent of any angle  $\theta^\circ$  is given in short by the ratio:

$$\text{Tan } \theta^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$$

Therefore in order for you to remember the definitions of sine, cosine and tangent is to use the abbreviation SOHCAHTOA. This means that sine(S) is opposite (O) over Hypotenuse (H), Cosine (C) is Adjacent (A) over Hypotenuse (H) and tangent is Opposite (O) over Adjacent (A).

Thus, the summary can be given as:

SO	TO	CA
H	A	H

### Examples

- In triangle ABC,  $AB = 3\text{cm}$ ,  $BC = 4\text{cm}$  and  $AC = 5$ . Find
  - sin A
  - cos A
  - tan A.

### Solution

You have to set up the plan to represent the information given in the questions as follows in Figure 2.

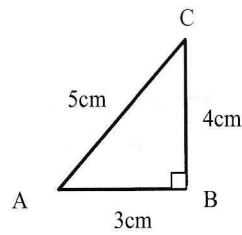


Figure 2.2

(a) Sin A

$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{4}{5}$$

$$\therefore \sin A = \frac{4}{5} = 0.8$$

(b) cos A :

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{3}{5}$$

$$\therefore \cos A = \frac{3}{5} = 0.6$$

(c) tan A:

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB} = \frac{4}{3}$$

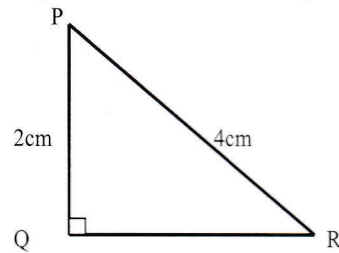
$$\therefore \tan A = \frac{4}{3} = 0.75$$

2. In  $\Delta PQR$ , angle PQR is a right angle. If  $RP = 4\text{cm}$  and  $PQ = 2\text{cm}$ . Find

- (a) QR
- (b)  $\tan R$
- (c)  $\cos R$ .

**Solution**

Set up of Figure 2.3 to represent the information given in the question.

**Figure 2.3**

(a) By applying the Pythagoras theorem

$$(QR)^2 + 2^2 = 4^2$$

$$(QR)^2 = 16 - 4$$

$$(QR)^2 = 12$$

$$QR = \sqrt{12} = 2\sqrt{3}$$

$$\therefore QR = 2\sqrt{3}$$

$$\begin{aligned} \text{(b) } \tan R &= \frac{\text{Opposite}}{\text{Adjacent}} \\ &= \frac{PQ}{QR} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ rationalizing the denominator;} \\ &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \end{aligned}$$

$$\therefore \tan R = \frac{\sqrt{3}}{3}$$

$$\text{(c) } \cos R = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos R = \frac{\sqrt{3}}{2}$$

**Activity 1**

Now do the following exercise after you have studied examples above.

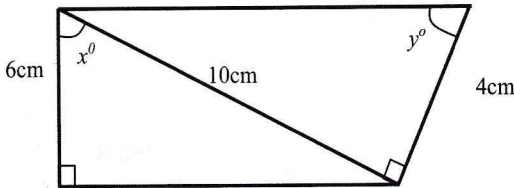
- Given that in  $\triangle ABC$ ,  $AB = 8\text{cm}$ ,  $AC = 15\text{cm}$  and  $BC = 17\text{cm}$ . Find the value of each of the following.
  - $\sin C$
  - $\tan C$
  - $\cos C$
- In triangle LMN,  $LM = 10\text{cm}$ ,  $MN = 6\text{cm}$  and  $LN = 8\text{cm}$ . Find
  - $\tan M$
  - $\sin M$
  - $\cos M$
- If  $5\cos\theta^\circ = 3$  and  $\theta^\circ$  is an acute angle. Find
  - $\tan\theta^\circ$
  - $\sin\theta^\circ$
- Given that ABC is a right-angled triangle with a right angle at B. Find in terms of  $a$ ,  $b$  and  $c$ , if  $AB = c$ ,  $BC = a$  and  $AC = b$ . Find the value of each of the following:
  - $\sin C$
  - $\cos C$
  - $\tan C$
  - $\frac{\sin \hat{C}}{\cos \hat{C}}$
- 

Figure 2.4

Find

- $\cos x^\circ$  and
- $\cos y^\circ$

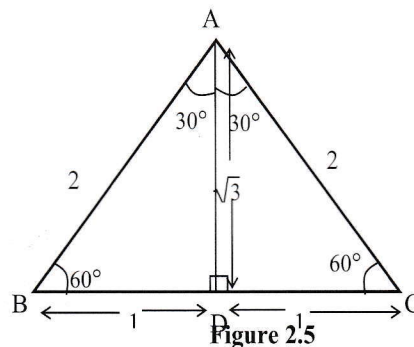
Compare your answers with those given at the end of the unit.



## Special Angles

### Trigonometric Ratios for Special Angles

The trigonometric ratios for special triangles can be found if you will consider the equilateral triangle ABC as shown in Figure 2.5 below.



From Figure 18 you will see that AD bisects BC at D, then  $BD = DC$  because of bisection. By Pythagoras theorem AD can be found as:

$$(AD)^2 + (CD)^2 = (AC)^2$$

$$(AD)^2 + (1)^2 = 2^2$$

$$(AD)^2 = 4 - 1$$

$$(AD)^2 = 3$$

$$\therefore AD = \sqrt{3}$$

(i) Consider Angle  $30^\circ$

$$\sin 30^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BD}{AB} = \frac{1}{2}$$

$$\therefore \sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BD}{AD} = \frac{1}{\sqrt{3}}, \text{ By rationalising denominator}$$

$$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\therefore \tan 30^\circ = \frac{\sqrt{3}}{3}$$



(ii) Consider Angle  $60^\circ$

**sin  $60^\circ$**

$$\sin 60^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$$

$$\therefore \sin 60^\circ = \frac{\sqrt{3}}{2}$$

**cos  $60^\circ$**

$$\cos 60^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BD}{AB} = \frac{1}{2}$$

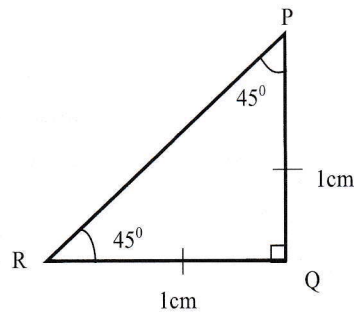
$$\therefore \cos 60^\circ = \frac{1}{2}$$

**tan  $60^\circ$**

$$\tan 60^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AD}{BD} = \frac{\sqrt{3}}{1}$$

$$\therefore \tan 60^\circ = \sqrt{3}$$

Also the trigonometric ratios for special angles can be found by considering the isosceles triangle PQR as shown in Figure 2.6.



**Figure 2.6**

By using the Pythagoras theorem, the third side (hypotenuse) you can find it as follows:-

$$(PQ)^2 + (QR)^2 = (PR)^2$$

$$1^2 + 1^2 = (PR)^2$$

$$(PR)^2 = 1 + 1$$

$$(PR)^2 = 2$$



$$PR = \sqrt{2}$$

From Figure 19, you will be able to find the values of the sine, cosine and tangent of  $45^\circ$  as follows:

$$\sin 45^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{1}{\sqrt{2}}$$

Rationalising the denominator

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\therefore \sin 45^\circ = \frac{\sqrt{2}}{2}$$

**cos  $45^\circ$**

$$\cos 45^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{RQ}{PR} = \frac{1}{\sqrt{2}}$$

rationalising the denominator

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

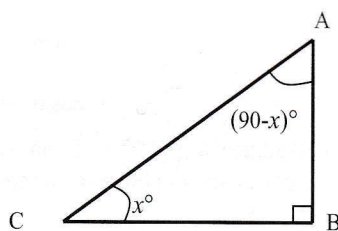
$$\therefore \cos 45^\circ = \frac{\sqrt{2}}{2}$$

**tan  $45^\circ$**

$$\tan 45^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{PQ}{RQ} = \frac{1}{1} = 1$$

$$\therefore \tan 45^\circ = 1$$

The trigonometric ratios for special angles can also be found if you will consider the two complementary angles as shown in Figure 2.7.



**Figure 2.7**

From Figure 7, you will see that angle C and angle A are complementary angles. Thus, complementary angles are the angles which when summed give  $90^\circ$ .



$$\text{i.e. } m(\widehat{BCA}) + m(\widehat{CAB}) = 90^\circ$$

If  $m(\widehat{BCA}) = x^\circ$  and  $m(\widehat{CAB}) = 90 - x$  then,

$$x^\circ + (90 - x)^\circ = 90^\circ \text{ (complementary Angles).}$$

Therefore, you will find the sine and cosine of these angles as follows:-

$$\begin{aligned} \sin x^\circ &= \frac{AB}{AC} \quad \text{and} \quad \sin (90 - x)^\circ = \frac{BC}{AC} \\ \cos x^\circ &= \frac{BC}{AC} \quad \text{and} \quad \cos (90 - x)^\circ = \frac{AB}{AC} \end{aligned}$$

From these ratios you will discover that sine of one angle is equal to the cosine of its complementary angle.

Thus,

$\sin x^\circ = \cos (90 - x)^\circ$
$\cos x^\circ = \sin (90 - x)^\circ$

By using these identities you will see that;

$$\begin{aligned} \sin 30^\circ &= \cos 60^\circ = \frac{1}{2} \\ \cos 30^\circ &= \sin 60^\circ = \frac{\sqrt{3}}{2} \\ \sin 45^\circ &= \cos 45^\circ = \frac{\sqrt{2}}{2} \end{aligned}$$

because each pairs of these angles are complementary to each other.

Also if  $x^\circ = 90^\circ$ , BC becomes zero and  $AC = AB$ . In this case you will see that;

$$\sin 90^\circ = \frac{AB}{AC} = 1, \text{ hence}$$

$$\cos 0^\circ = 1$$

And hence  $\sin 90^\circ = 1$  and  $\cos 0^\circ = 1$ . The trigonometric ratios for special angles above are summarised by Table 1 below.



Table 1: Trigonometric ratios for special angles

$x^\circ$	$\sin x^\circ$	$\cos x^\circ$	$\tan x^\circ$
$0^\circ$	0	1	0
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^\circ$	1	0	Undefined

### Problems Involving Trigonometric ratios of special angles

By using the angles in Table 1 the sine, cosine and tangent of angles  $30^\circ$  and  $60^\circ$  you can find them as follows:

#### Examples

1. Find the exact value of  $2\sin 60^\circ + \cos 30^\circ$

#### Solution

By using data from Table 1 you see that  $2\sin 60^\circ + \cos 30^\circ$  will be:

$$\begin{aligned} 2 \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} &= \sqrt{3} + \frac{\sqrt{3}}{2} \\ &= \frac{2\sqrt{3} + \sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \end{aligned}$$

2. Without using tables evaluate the following

(a)  $\frac{\cos 60^\circ \times \sin 60^\circ}{\tan 60^\circ}$

#### Solution

$$\cos 60^\circ = \frac{1}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \text{ and}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\therefore \frac{\frac{1}{2} \times \frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{\sqrt{3}}{4} \div \sqrt{3}$$

$$\therefore \frac{\sqrt{3}}{4} \times \frac{1}{\sqrt{3}} = \frac{1}{4}$$



$$(b) \quad \sin 60^\circ(\tan 30^\circ - \cos 30^\circ)$$

**Solution**

By using table 1 above you will see that:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{\sqrt{3}}{3} \quad \text{and}$$
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{\sqrt{3}}{2} \left( \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{2} \right)$$

Opening the bracket:

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$\frac{3}{6} - \frac{3}{4} = \frac{6-9}{12} = \frac{-3}{12}$$

$$\therefore \sin 60^\circ(\tan 30^\circ - \cos 30^\circ) = \frac{-1}{4}$$

$$c) \quad \frac{\sin 90^\circ}{\tan 30^\circ}$$

**Solution**

By using the summary of special angle of table 1 above you will have:

$$\sin 90^\circ = 1 \quad \text{and} \quad \tan 30^\circ = \frac{\sqrt{3}}{3}$$
$$\therefore \frac{1}{\frac{\sqrt{3}}{3}} = 1 \div \frac{\sqrt{3}}{3} = 1 \times \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore = \frac{3\sqrt{3}}{3} = \sqrt{3} \text{ Answer.}$$

1. Without using tables evaluate the following

$$\frac{\sin 120^\circ + \cos 135^\circ}{\cos 150^\circ + \cos 120^\circ}$$

**Solution**

$\sin 120^\circ = \sin (180^\circ - 120^\circ) = \sin 60^\circ$  since  $60^\circ$  and  $120^\circ$  are supplementary angles, therefore you can say that sine, cosine or tangent of obtuse angles are equal to the sine, cosine or tangent of the corresponding supplementary angles.

Thus,

$$\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$$

$$\begin{aligned} \text{then, } \frac{\sin 120^\circ + \cos 135^\circ}{\cos 150^\circ + \cos 120^\circ} &= \frac{\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{3}}{2} - \frac{1}{2}} = \left( \frac{\sqrt{3} - \sqrt{2}}{2} \right) \div \left( \frac{-\sqrt{3} - 1}{2} \right) \\ &= \frac{\sqrt{3} - \sqrt{2}}{2} \times \frac{2}{-\sqrt{3} - 1} = \frac{\sqrt{3} - \sqrt{2}}{-\sqrt{3} - 1} \\ &= \frac{\sqrt{2} - \sqrt{3}}{2} \times \frac{2}{\sqrt{3} + 1} = \frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} + 1} \end{aligned}$$

rationalising denominator

$$= \left( \frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} + 1} \right) \left( \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right) \text{ since } (\sqrt{3} - 1) \text{ is the}$$

rationalising factor

$$\begin{aligned} &= \frac{\sqrt{6} - \sqrt{2} - 3 + \sqrt{3}}{3 - 1} \\ &= \frac{\sqrt{6} + \sqrt{3} - \sqrt{2} - 3}{2} \end{aligned}$$

**Activity 2**

Attempt the following questions:



- Find the value of each of the following
  - $\frac{\sin 90^\circ}{\cos 0^\circ}$
  - $\frac{\sin 60^\circ}{\cos 60^\circ}$
  - $\frac{\sin 30^\circ}{\cos 30^\circ}$
- Evaluate the following with
- Find the exact value of each of the following.
  - $\sqrt{3} \tan 60^\circ$
  - $\sqrt{2} (\cos 45^\circ + \cos 60^\circ)$
- Simplify the following
  - $\sin 60^\circ (\tan 30^\circ - \cos 30^\circ)$
  - $\cos 60^\circ (\sin 30^\circ + \cos 30^\circ)$
- If  $\cos x^\circ = 0.5$  and  $x^\circ$  is an acute angle, find
  - $x$
  - $\tan x^\circ$
  - $\sin x^\circ$

Compare your answers with those given at the end of the unit

**Problems Involving Trigonometric Ratios****Tables of Trigonometric Ratios**

The values of trigonometric ratios can be read from the trigonometric tables just as it is done with logarithms. Consider that you have a table of sine as shown in Table 2.



**Table 2: Natural Sine**

Degrees	Minutes					Mean differences								
	0'	10'	20'	30'	40'	1	2	3	4	5	6	7	8	9
0°	0.0000	.00	...	..	.01	3	6	9	1	1	..	..	..	..
1°	.0175	17	...	..	40	3	6	9	2	5	..	..	..	..
...		.02	...	..	.02				1	1				
		.04		→	91				2	5				
36°	.005878		...			2	5	7						
...		.59	...	..	.59				9	1				
74°	.9613	01	...	..	72	1	2	2		2	5	5	6	7
...		.96	...	..	.96	..	...	..	3	4	..	..	..	..
		21		..	44	..	..	..	..	..	..	..	..	..
		...		..	...				..	..	..	..	..	..

Dear learner, the angle is read from the extreme left-hand column. For example, you are told to read  $\sin 36^\circ$  from the table, what you have to do here is to read the angle from extreme left hand column and read the corresponding value under the column labelled 0'. This gives you the value of 0.5878. Thus  $\sin 36^\circ$  is equal to 0.5878. This value is read from the table of the Natural Sines. Similarly, if you have to find or to read  $\sin 74^\circ 40'$ , first you have to read  $\sin 74^\circ$  on the extreme left hand in the table of Natural sines and in this row reads the corresponding value under 40' which will give the value of 0.9644. Thus  $\sin 74^\circ 40' = 0.9644$ . (Table 2)

Likewise to find  $\sin 74^\circ 48'$ , first you locate angle  $74^\circ$  on the extreme left hand column and in this row read the corresponding value under 40'.

Thus,  $\sin 74^\circ 40' = 0.9644$  and again in the same row read the corresponding value under 8' in the difference column. This gives the value of 6. Add it to the last digit.



That is  $\sin 74^\circ 40' = 0.9644$  and the corresponding value under 8' in the differences column gives 6. If you add you will get

$$\begin{array}{r} 0.9644 \\ + \quad 6 \\ \hline 0.9650 \end{array}$$

Hence,  $\sin 74^\circ 48' = 0.9650$

Dear learner, tables of cosines and tangents are read in exactly the same way as you read sine.

For cosine tables, the method is the same but the differences have to be **subtracted**. For example you are asked to find the value of  $\cos 64^\circ 13'$ . What you have to do here is that locate  $64^\circ$  in the extreme left hand column of the Natural cosine tables and read the corresponding value under 10' which gives 0.4358, then read corresponding value under 3 in the mean difference column, which gives 8 and hence subtract from 0.4358 thus,

$$\begin{array}{r} 0.4358 \\ - \quad 8 \\ \hline 0.4350 \end{array}$$

$\therefore$  hence  $\cos 64^\circ 13' = 0.4350$

Therefore you are given the tables of Natural Sine, Natural Cosines and Natural Tangents as shown in tables at the end of the unit.

#### Examples

1. By using trigonometric tables, find

(a)  $\cos 16^\circ 24'$

(b)  $\sin 58^\circ$

**Solution** (these are obtained directly from tables)

(a)  $\cos 16^\circ 24' = 0.9593$

(b)  $\sin 58^\circ = 0.8480$

2. Use mathematical tables to find the following

(a)  $\sin 20^\circ 14'$

(b)  $\cos 38^\circ 18'$

(c)  $\tan 48^\circ 20'$

**Solution**

(a)  $\sin 20^\circ 14' = 0.3453$

$$\begin{array}{r} + \quad 5 \\ \hline 0.3458 \end{array}$$



$$\therefore \sin 20^\circ 14' = 0.7848$$

$$(b) \quad \cos 38^\circ 18' = 0.7848$$

$$\therefore \cos 38^\circ 18' = 0.7848$$

$$(c) \quad \tan 48^\circ 20' = 1.1224$$

$$+ \frac{13}{1.1237}$$

$$\therefore \tan 48^\circ 20' = 1.1237$$

3. By using mathematical tables, find the value of each of the following:

(a)  $\cos 40^\circ 16'$

(b)  $\tan 38^\circ 28'$

**Solution**

(a)  $\cos 40^\circ 16'$

By using table of natural cosines

$$\cos 40^\circ 12' = 0.7638$$

$$- \frac{4}{0.7630}$$

$$\therefore \cos 40^\circ 16' = 0.7630$$

(b)  $\tan 38^\circ 28'$

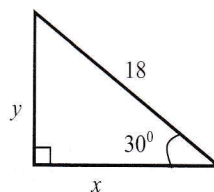
By using table of the natural tangents

$$\tan 38^\circ 24' = 0.7926$$

$$4' + \frac{19}{0.7945}$$

$$\therefore \tan 38^\circ 28' = 0.7945.$$

4. Find the values of  $x$  and  $y$  in Figure 2.8



**Figure 2.8**

**Solution**

- (i) The value of  $x$ .

$$\cos 30^\circ = \frac{x}{18} \Rightarrow x = 18 \times \cos 30^\circ$$

$$\text{but } \cos 30^\circ = 0.8660$$

$$\therefore x = 18 \times 0.8660$$



$$\therefore x = 15.588$$

(ii) The value of  $y$ .

$$\sin 30^\circ = \frac{y}{18} \Rightarrow y = 18 \times \sin 30^\circ$$

$$\text{but } \sin 30^\circ = 0.5$$

$$\therefore y = 18 \times 0.5$$

$$\therefore y = 9.$$



### Activity 3

Now answer the following questions:

- Use tables to find the value of each of the following:
  - $\sin 56^\circ$
  - $\tan 36^\circ$
  - $\cos 80^\circ$
- Find the value of each of the following:
  - $\cos 65^\circ 15'$
  - $\sin 37^\circ 11'$
  - $\tan 70^\circ 24'$
- By using tables, find the value of  $x$ .
  - $\cos x^\circ = 0.2034$
  - $\sin x^\circ = 0.5975$
  - $\tan x^\circ = 1.5000$
- Find the value of  $\theta$  in each of the following:
  - $\cos \theta^\circ = \frac{2}{5}$
  - $\tan \theta^\circ = \frac{1}{3}$
  - $\sin \theta^\circ = \frac{1}{8}$



5. Find the value of  $x$  and  $y$  in the Figure 2.9

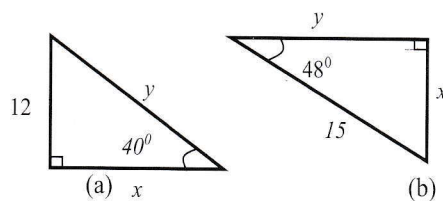


Figure 2.9

Compare your answers with those given at the end of the unit.

## Angles of Depression and Elevation

Dear learner, consider an object situated at a certain point at B as shown in Figure 2.10

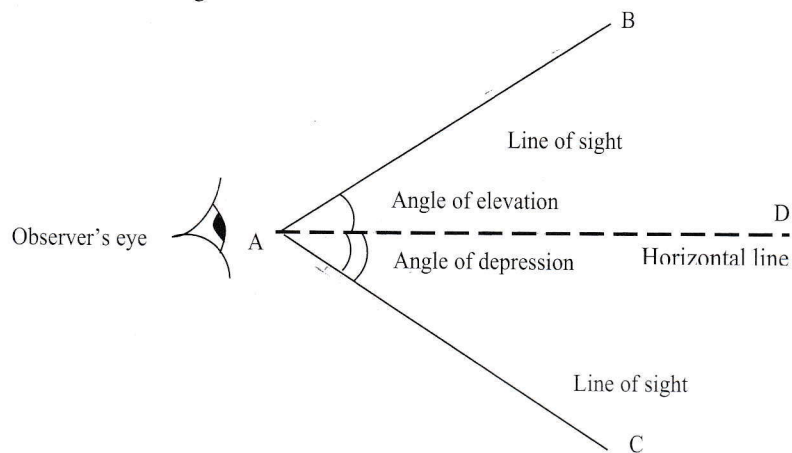


Figure 2.10

From Figure 11, you will see that the line joining the observer's eye at A is called the line of sight for B. The object to be observed may be above or below the horizontal line. If the object is above the horizontal line, then the angle formed between the line of sight and the horizontal line is called the angle of *elevation*. If the line of sight is below the horizontal, then the angle formed between the line of sight and the horizontal is called the angle of *depression*. Thus, from Figure 11,  $m(\text{BAD})$  is the angle of Elevation and  $m(\text{CAD})$  is the angle of depression.

The knowledge of Angle of Depression and angle of Elevation is very essential in solving various trigonometric problems; therefore I



hope you will apply the knowledge of these angles to solve various problems which are related to trigonometric problems.

### Problems Involving Angles of Depression and Elevation

#### Examples

1. From the top of a tower, the angle of depression of a point on the ground 10m away from the base of the tower is  $60^\circ$ . How high is the tower.

#### Solution

Sketch a diagram to represent the information given.

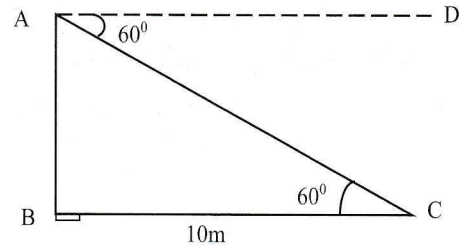


Figure 2.11

Let A represent the top of the tower, C the point observed and B the base of the tower as shown in figure 24 above.

Therefore, in triangle ABC.

$$\tan 60^\circ = \frac{AB}{10}$$

$$\tan 60^\circ = \frac{h}{10} \Rightarrow h = 10 \times \tan 60^\circ$$

But from the table of Natural tangents,  $\tan 60^\circ = 1.7321$

$$\text{Thus, } h = 10 \times 1.7321 = 17.32$$

**$\therefore$  the height of the tower is 17.32**

2. The length of a shadow of a 16m tree is 8m. What is the size of angle of elevation of the sun?

#### Solution

Sketch/draw the plan so as to represent the information



given from the question.

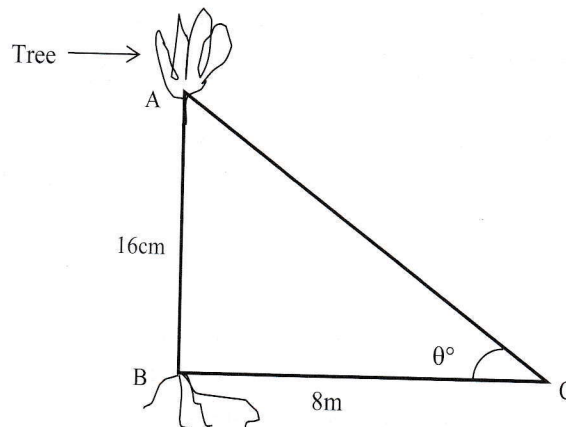


Figure 2.12

By applying the tangent of the angle  $\theta^\circ$  in Figure 25 you will find:

$$\tan \theta^\circ = \frac{16}{8} \text{ since } \tan \theta^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$$

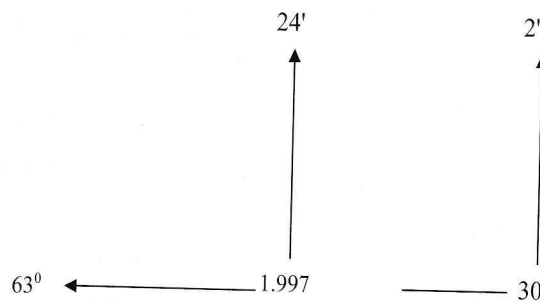
$$\tan \theta^\circ = 2$$

tangent inverse of 2 is equal to  $\theta^\circ$  (from mathematical tables of inverse tangent).

$$\text{Thus, } \tan^{-1} 2 = \theta^\circ$$

$$\theta^\circ = \tan^{-1} 2 = \underline{63^\circ 26'}$$

That is, you are required to check from the table of Natural tangents the value which is equal or approximately to 2 and then read the corresponding angle on the extreme left hand column and the corresponding minutes at the top from the approximate number as shown below.





But to get 2, you remain with 30, then read the corresponding value from this row in the means difference column, which gives 2'

$$\begin{array}{r} \text{Thus, } 2.0000 \\ - 1.9970 \\ \hline 30 \end{array}$$

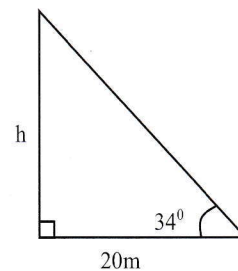
$$\begin{array}{r} \text{Hence, the angle is } 63^\circ 24' \\ + 2' \\ \hline 63^\circ 26' \end{array}$$

$\therefore$  The angle of elevation is  $63^\circ 26'$

3. Find the height of a tower if the angle of elevation of the top is  $34^\circ$  at a point 20m from its foot.

**Solution**

You have to set up the drawing to illustrate the information given from the question.



**Figure 2.13**

By applying the tangent of angle  $34^\circ$  in Figure 2.13, you will get

$$\tan 34^\circ = \frac{h}{20}, \text{ since}$$

$$\tan \theta^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$h = 20 \times \tan 34^\circ$$

but  $\tan 34^\circ = 0.6745$  from the table of Natural tangents, table 1.3

$$h = 20 \times 0.6745$$

$$h = 13.49\text{m}$$

$\therefore$  The height of the tower is 13.49m.



4. The angle of elevation of the top of a tree from a point on the ground 30m from the base of the tree is  $37^\circ$ . Find the height of the tree.

#### Solution

You have to set up the drawing (diagram) to illustrate the information given from the question.

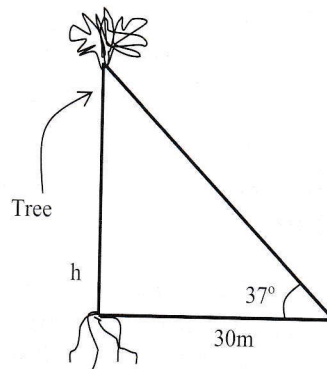


Figure 2.14

By applying the tangent of angle  $37^\circ$  in Figure 27, you will get

$$\tan 37^\circ = \frac{h}{30} \text{ since } \tan \theta^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$h = 30 \times \tan 37^\circ$$

But  $\tan 37^\circ = 0.7536$  from the table of Natural tangents.

$$h = 30 \times 0.7536$$

$$h = 22.608$$

$\therefore$  The height of the tower is 22.61m.



#### Activity 4

Now attempt the following questions:

1. A man whose eye is 120cm above the ground is standing 8m from a tree 7m tall. What is the angle of elevation of the top of the tree from his eye?
2. The length of shadow of a tree is 8m. What is the size of the angle of elevation if its hypotenuse is 8.7m?
3. Find the height of a tower if the angle of elevation of the top is  $54^\circ$  at a point 30m from its foot.
4. The angle of elevation of the top of a building 24m high is observed from the top and from the bottom of a vertical



ladder, and found to be  $45^\circ$  and  $60^\circ$  respectively. Find the height of the ladder.

5. A is 289m from C on a bearing of  $N32^\circ W$ , B is 450m from C on bearing of  $N58^\circ E$ . Find the distance from A to B.
6. When the angle of elevation of the sun is  $55^\circ$  a tower casts a shadow 20m long. Find the height of the tower.

*Compare your answers with those given at the end of the unit.*

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## Unit Reflection



Dear learner, you are through with this unit, what are the new concepts you have learned in this unit and how can you apply these concepts in your daily life?

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## Unit Assignment



*Now attempt the following questions:*

1. In PQR, angle PQR is a right angle. If  $RP = 4\text{cm}$  and  $PQ = 2\text{cm}$   
Find (i) QR  
(ii)  $\tan R$   
(iii)  $\cos R$
2. Find the value of the following without using tables
  - (a)  $\frac{\cos^2 45^\circ + \sin^2 60^\circ}{\tan^2 60^\circ}$
  - (b)  $\frac{\sin 45^\circ \times \sin 60^\circ}{\sin 30^\circ}$
3. Given that  $\cos y^\circ = \frac{15}{17}$ . Find the value of
  - (i)  $\tan y^\circ$
  - (ii)  $\sin y^\circ$



4. If  $5\sin C = 3$ . Find
  - (i)  $\cos C$
  - (ii)  $\tan C$
5. Without using tables find the value of each of the following.
  - (a)  $\tan 30^\circ (\tan 60^\circ + \sin 60^\circ)$
  - (b)  $\cos 45^\circ (\sin 45^\circ + \cos^2 30^\circ)$
6. By using tables, Find the value of the following:
  - (a)  $\sin 38^\circ 18'$
  - (b)  $\cos 40^\circ 11'$
  - (c)  $\tan 40^\circ 12'$
7. By using the area of Trapezium, prove that  $a^2 + b^2 = c^2$
8. In triangle ABC,  $AB = 2\text{cm}$ ,  $BC = 8\text{cm}$  and  $M(\angle ABC) = 90^\circ$  find
  - (i) AC
  - (ii)  $\sin C$
  - (iii)  $\cos A$ .
9. C is 5km from A on a bearing of  $N32^\circ W$ , B is 12km from A on a bearing of  $N58^\circ E$ . Find the distance from B to C.
10. P and Q are two legs on the ground level, and both lie due west of a Flagstaff. The angle of elevation of the top of the flagstaff from P is  $45^\circ$  and from Q is  $60^\circ$ . If P is 24m from the foot of the Flagstaff, Find PQ.

### Key Answers for Activities

#### Activity 1



1. (i)  $\frac{8}{17}$  (ii)  $\frac{8}{15}$  (iii)  $\frac{15}{17}$
2. (i)  $\frac{8}{6}$  (ii)  $\frac{8}{10}$  (iii)  $\frac{6}{10}$
3. (i)  $\frac{4}{3}$  (ii)  $\frac{4}{5}$
4. (i)  $\frac{c}{b}$  (ii)  $\frac{a}{b}$  (iii)  $\frac{c}{a}$  (iv)  $\frac{c}{a}$



5. (i)  $\frac{\sqrt{116}}{29}$  (ii)  $\frac{6}{10}$

**Activity 2**

1. (a) 1 (b)  $\sqrt{3}$  (c)  $\frac{\sqrt{3}}{3}$

2. (a) 0 (b)  $\frac{\sqrt{2}}{4}$

3. (a) 3 (b)  $\frac{2+\sqrt{2}}{2}$

4. (a)  $-\frac{1}{4}$  (b)  $\frac{1+\sqrt{3}}{4}$

5. (i)  $x = 60^\circ$  (ii)  $\sqrt{3}$  (iii)  $\frac{\sqrt{3}}{2}$

**Activity 3**

1. (a) 0.8290

(b) 0.7265

(c) 0.1736

2. (a) 0.4187

(b) 0.6044

(c) 2.8083

3. (a)  $78^\circ 16'$

(b)  $36^\circ 41'$

(c)  $56^\circ 19'$

4. (a)  $66^\circ 25'$

(b)  $18^\circ 26'$

(c)  $7^\circ 11'$

5. (a)  $x = 14.30$

$y = 18.67$

(b)  $x = 11.15$

$y = 10.05$



**Activity 4**

1.  $35^{\circ}57'$
2.  $63^{\circ}26'$
3. 41.29m
4. 10.14
5. 534.9m 28.56m



## Unit 3

---

### Demonstrating Geometrical Transformation

#### Introduction

Dear learner, I hope you once looked at yourself in the mirror and saw your image behind it.

You may have seen the driver using the side mirror so as to see if there are any cars coming behind and in front of him/her when he/she wants to overtake. This image from the mirror is due to *Reflection*.

Again look at the watch on your wrist, do you see how the arrow moves? This arrow rotates from one point to another.

Likewise you have as well seen people move things like bricks, tables, chairs from one position to another, this is known as *Translation*.

In this unit you will be introduced to the concept of Transformation. The unit consists of three sections which are Reflection, Rotation, translation and enlargement; and combined transformation.

I hope you will enjoy studying this unit the way you did in the previous units.

#### Learning Outcomes



Upon completion of this unit you should be able to:

- *Work* with reflection and rotation translation and enlargement and combined transformation;
- *Explain* the difference between reflection and rotation, translation and enlargement
- *Apply* the knowledge of transformation to solve problems related to daily life activities.



## Reflection

Dear learner, please take a mirror and see a clock behind you through the mirror. What time does the real clock show and what time the image in the mirror show? (Use a clock with arrows for this exercise).

Also visit a barber shop, observe a person who is dressing the hair through the mirror, what is the position of the cutting machine to a real person? And what is the position of the cutting machine of the image?

Reflection is defined as a transformation that gives a "mirror" image across a line, called the reflection line.

If  $P$  is a point not on line  $L$ , then its image in  $L$  is the point  $P'$  such that  $PP'$  is perpendicular to line  $L$  and  $PX = P'X$  if line  $PP'$  meets line  $L$  at  $X$ .

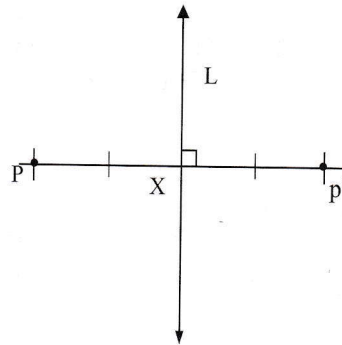


Figure 3.1

**Note:** Perpendicular means making  $90^\circ$  to each other.

After you have familiarized yourself with what is reflection, let us see how reflection can be done using any paper or tracing paper.

### Example 1

Reflect the letter  $F$  on the line  $L$  indicated in Figure 3.2.

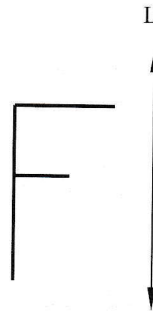
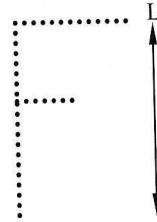


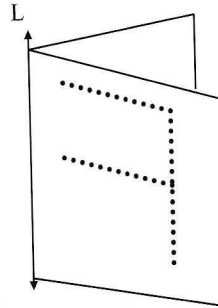
Figure 3.2

**Solution**

1. Draw your figure on tracing paper or other paper. Make holes using office pins on letter F so that hole can be seen after folding. Figure 4.

**Figure 3.3**

2. Fold your piece of paper along line L by turning up the side of the letter F. Figure 3.4.

**Figure 3.4**

3. Fold your piece of paper with the letter F to the other side without letter, you will see your holes which were used to make the letter F. Using your sharp pencil put marks through the holes to make the image of your letter F after folding. See Figure 3.5.

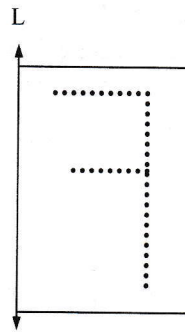


Figure 3.5

4. Then unfold your piece of paper to get the letter F and its image on the same piece of paper

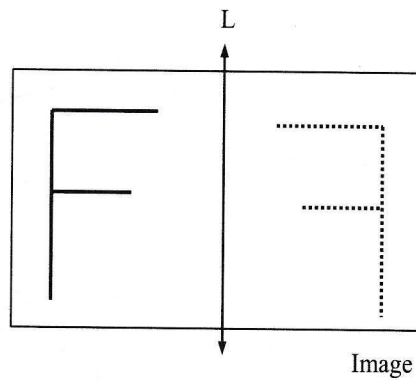


Figure 3.6

**Example 2**

Let  $ABC$  be any triangle and line  $L$  be line of reflection of the given triangle (Figure 3.7).

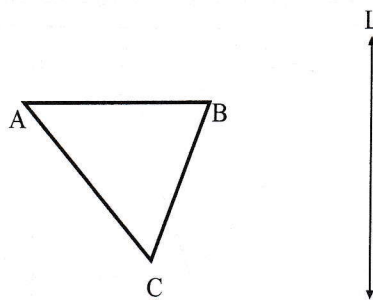


Figure 3.7

**Solution**

Follow the steps below to get an image of triangle  $ABC$ .



- (1) Trace (draw) triangle ABC on a piece of paper or tracing paper and make hole with a pin on triangle ABC.

- (2)

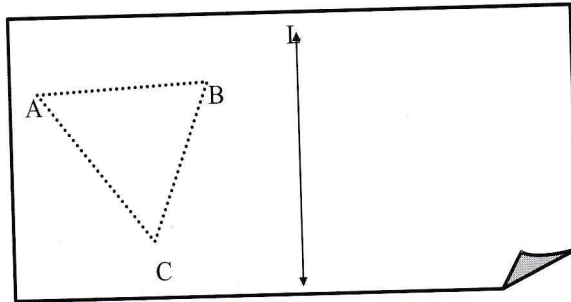


Figure 3.8

- (3) Fold your piece of paper along the line L (Figure 3.9).

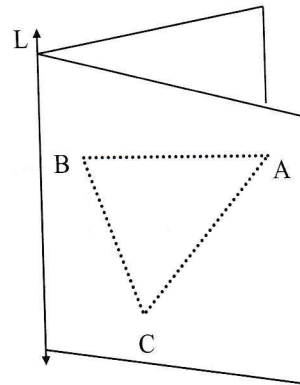
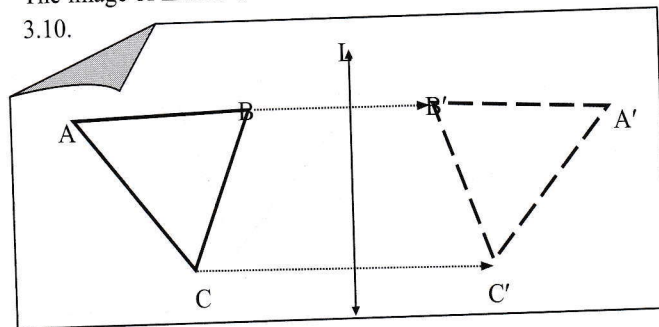


Figure 3.9

Using the holes made on triangle ABC with your pencil, put marks on the other side of the folded paper to produce the image of the triangle.

The image of  $\triangle ABC$  when reflected on L will be as shown in Figure 3.10.





### Reflection and Co-ordinate Geometry

Dear learner, you have been making a reflection of figures, but when these figures are drawn in co-ordinate axes, we are dealing with reflection of points which makes such figure:

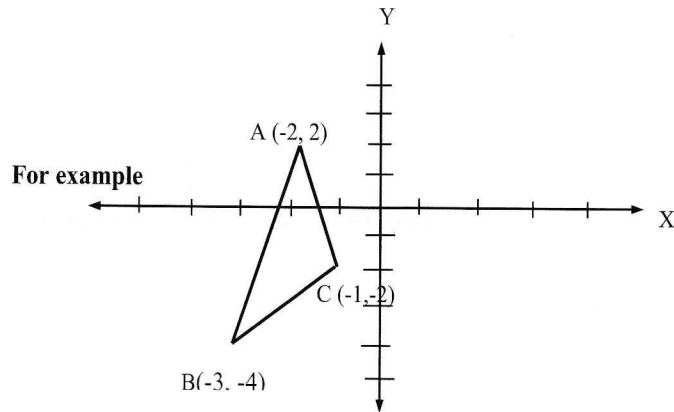


Figure 3.11

If triangle ABC is reflected on line  $x = 0$  that is the  $y$ -axis, the coordinate of A(-2, 2) changes to A'(2, 2) and B(-3, -4) changes to (3, -4) and coordinate C(-1, -2) changes to (1, -2).



Note that on reflection  $y$ -axis the value of  $x$  co-ordinates changes its sign and Figure 39 changes to Figure 3.12.

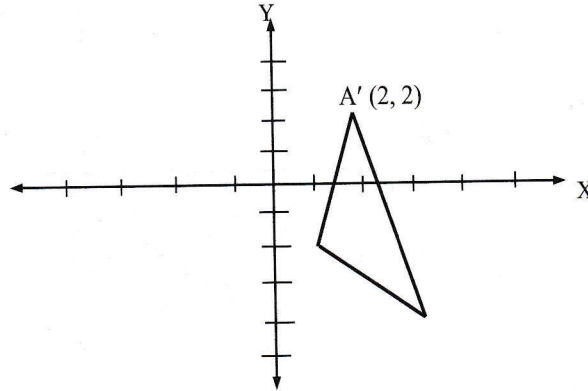


Figure 3.12

After learning about reflection of figures and co-ordinates in a plane now you can discuss the following questions.



## Activity 1



1. Reflect the figure below in the  $y$  and  $x$  lines.

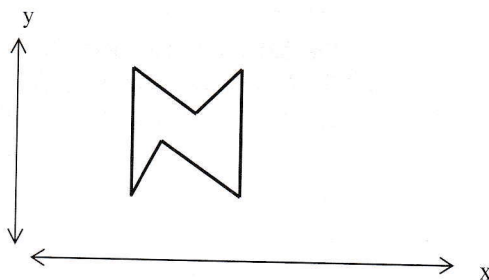


Figure 3.13

2. Find the image of the following figure on mirror  $M$ .

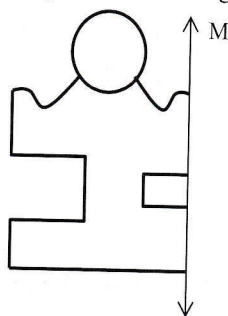


Figure 3.14

3. Give the co-ordinates of the images of the following points under reflection in line  $y = 0$ .  
(a)  $(3, -1)$       (b)  $(-6, -3)$
4. If  $(2, 1)$  is the image of  $(1, 2)$  draw the reflecting line.

*Compare your answers with those given at the end of the unit.*

## Rotation

Dear learner let us now discuss rotation as another transformation which is common in our daily life. Consider when you go to the tailor for making your new skirt or trouser, when you try it on the first time your tailor may want to see if it fits as it was planned. In so doing she/he asks you to turn around so that she/he can be able to see parts of your body and how the skirt fits you. In so doing you will be turning around or rotating. Also observe the tailor when designing and cutting the dress. What does the tailor do? How? I hope you will see a tailor turning the cloth in different positions. So

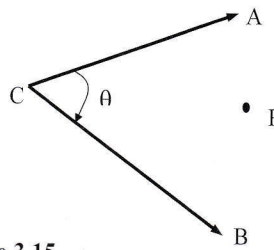


the application of rotation enables you to see the hidden part of the object.

Similarly, in mathematics we have rotation. We define rotation as transformation associated with a turning motion through a specified directed angle around a fixed point, called the centre and we denote rotation by  $R_\theta$ . ( $\theta^\circ$  angle of rotation).

As we learned in the last two sections rotation can also be done using tracing paper. In rotation instead of moving a point, we turn the plane on which the point is lying or positioned.

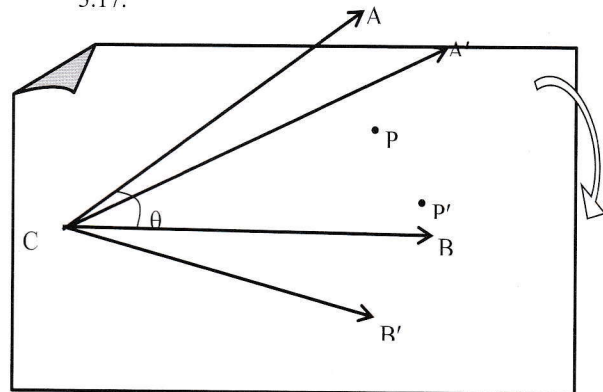
**For example,** Consider point P in the plane.



**Figure 3.15**

If you rotate point P in clock-wise direction through angle  $\theta$  do the following:

1. Trace Figure 3.15 on tracing paper.
2. Turn your line CA until it reaches line CB, then using a pin, make a hole on your tracing paper at point p to get of p' after rotation. See Figure 3.16 and Figure 3.17.



**Figure 3.16**

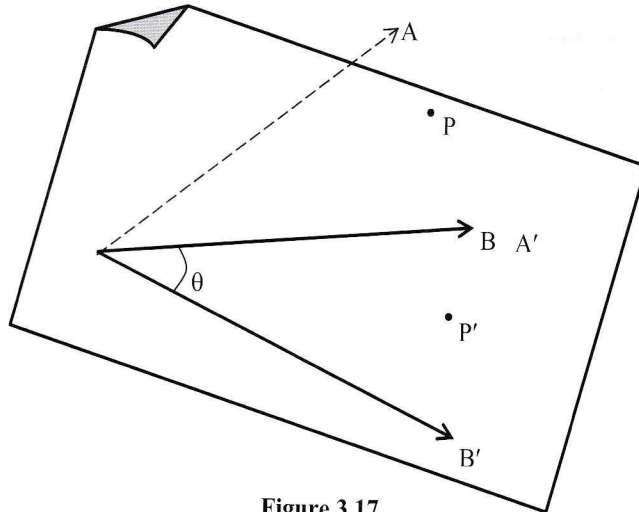


Figure 3.17

That is how a point in a plane can change its position by rotation. But all geometrical figures are made of points, therefore to rotate a figure in a plane you need to rotate as many points as you can on that figure, and then you draw the figure.

**Directions of Rotation**

In rotation we have two common direction of rotation. We have clockwise and anti-clock wise directions. Clockwise direction follows the direction of a common clock in counting time or moving from left to right.

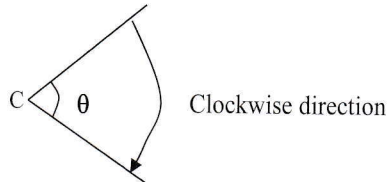


Figure 3.18

Anti-clockwise direction goes in opposite direction of a common clock in counting time or moving from right to left.

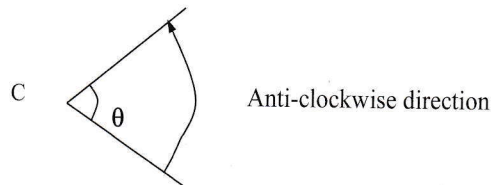
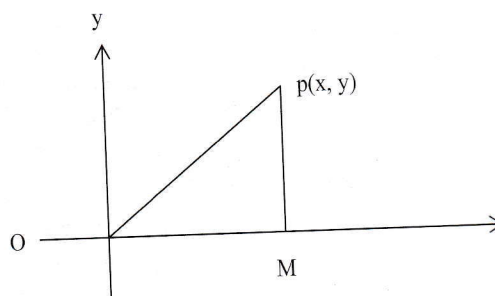


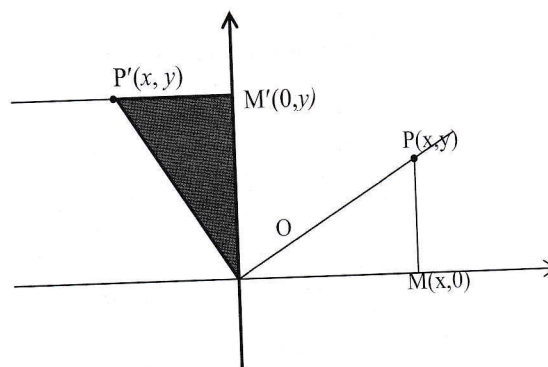
Figure 3.19

**Example 1**

Find the image of triangle OPM under rotation of  $90^\circ$  in anti-clockwise direction.

**Figure 3.20****Solution**

Trace your triangle OPM or tracing paper, fix the centre O and rotate your triangle  $90^\circ$  anti-clockwise. Then your solution will be as follows.

**Figure 3.21**

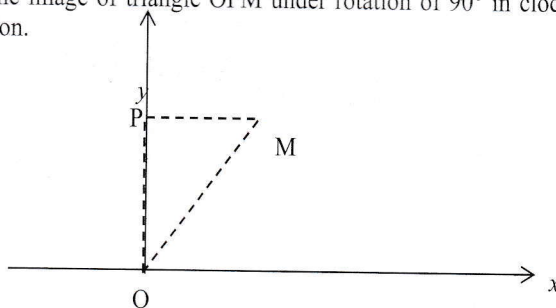
After rotation through  $90^\circ$  anticlockwise, the position of triangle OPM will be triangle  $OP'M'$  (Figure 3.21).

**Rotation by Using a Pair of Compasses**

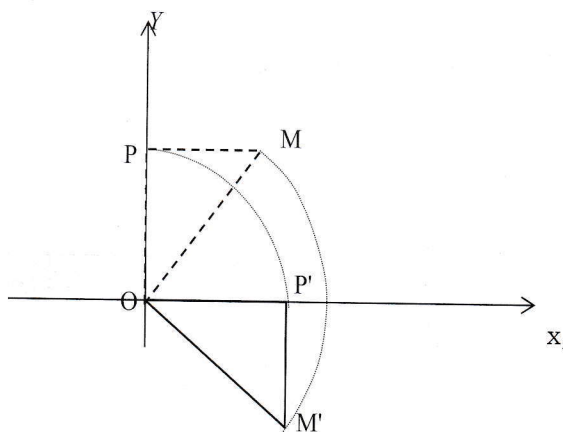
You can find the image of a given figure by using a pair of compasses.

**Example 3**

Find the image of triangle OPM under rotation of  $90^\circ$  in clockwise direction.

**Figure 3.22****Solution**

- (1) Use your pair of compasses with O as the centre of rotation in clockwise direction.

**Figure 3.23**

- (2) Measure angle POP' to be  $90^\circ$   
 (3) Measure angle MOM' to be  $90^\circ$   
 (4) Join OP', OM' and P'M' to get the image.

The reflection of point  $A(x,y)$  in the line  $y=x$  is  $A'(y,x)$ .

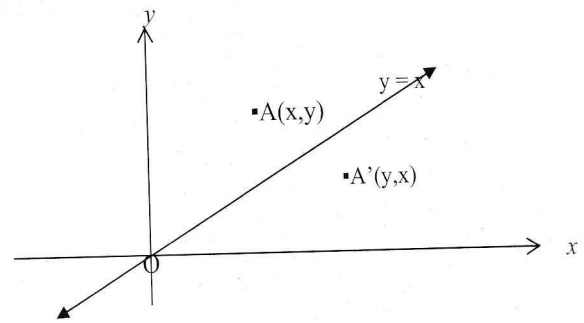


Figure 3.24

Also the reflection of the point  $B(a, b)$  in the line of  $y = -x$  is  $B'(-b, -a)$

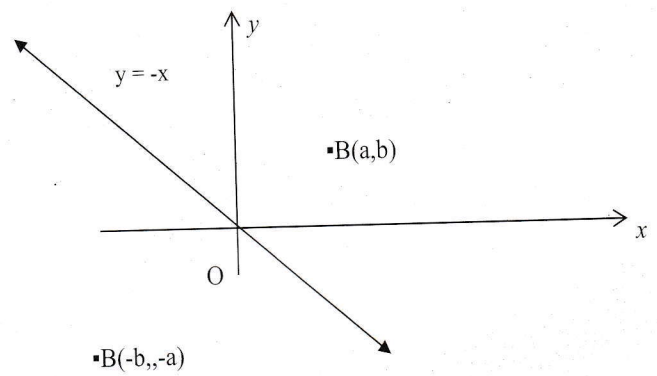


Figure 3.25



### Activity 2

Now do the following questions:

- Find the image of the following figure under  $R_{180}$  in anti-clockwise direction with centre  $O$ .

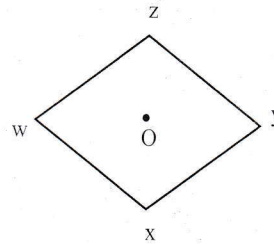


Figure 3.26

- Find the co-ordinates of the images of the following points under  $R_{180}$  (clockwise direction). In this question remember to draw  $x - y$  plane before rotation and use origin as centre of rotation.
  - $(3, -1)$
  - $(-6, -3)$
  - $(-4, 2)$

Compare your answers with those given at the end of the unit

## Translation and Enlargement

### Translation

Dear learner, I think you are very familiar with the word translator, a person who delivers the same message from one language to another without changing meaning. Similar meaning is used in mathematics to explain the change of position of a point from one station to another station, and we define **Translation** as a transformation of an object which is associated with a sliding motion of a specified distance and direction, without any turning. The distance and direction is usually indicated with an arrow or **directed line segment**.

### Example 1

Let  $P$  be any point in the **plane** and the arrow  $\overrightarrow{AB}$  is direction of translation of point  $P$  (Figure 3.27) (plane means any surface, for this case horizontal surface).

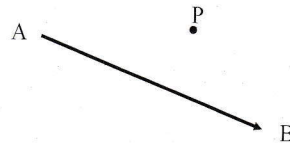


Figure 3.27



To translate point  $P$  along the arrow  $\overrightarrow{AB}$ , follow the following steps.

- 1) Take tracing material (any material which can allow you to see through). Trace the point  $P$  and arrow  $\overrightarrow{AB}$  on the tracing material (Figure 28).
- 2) Extend the arrow  $\overrightarrow{AB}$  in the direction of  $B$  on the original paper and not on the tracing paper (Figure 3.28).

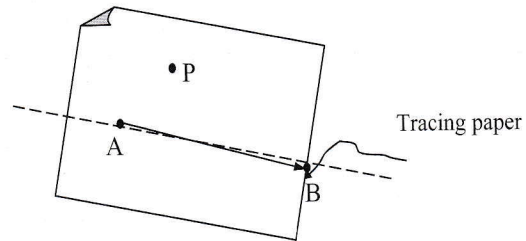


Figure 3.28

- 3) Make a hole at  $P$ ,  $A$  and  $B$  using a pin (office pin) then move your tracing paper along the extended line until point  $A$  reaches point  $B$ , then make a hole again through point  $P$  on tracing paper to get point  $P'$  (Figure 3.29).

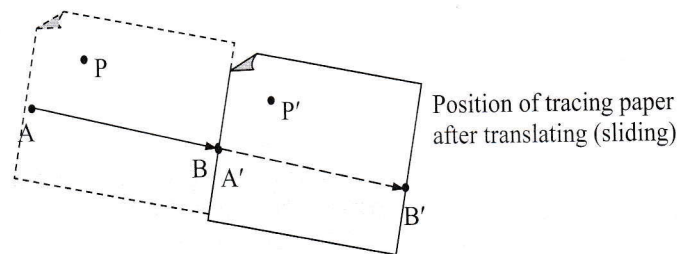


Figure 3.29

Point  $P'$  is the image of point  $P$  after translation along arrow  $\overrightarrow{AB}$  and can be seen as follows (Figure 30)

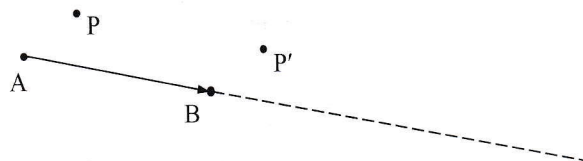
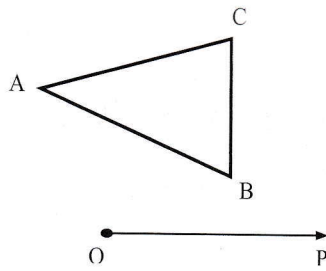


Figure 3.30

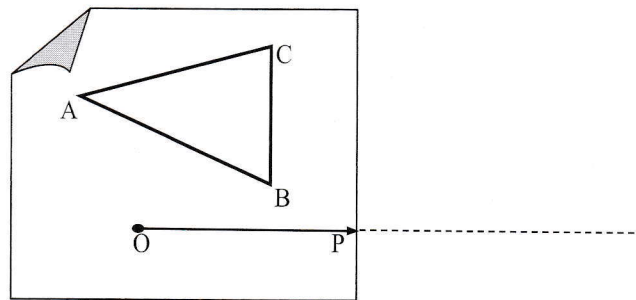
**Example**

Translate the following triangle ABC in the indicated arrow, (Figure 3.31).

**Figure 3.31****Solution**

Use the same steps as in example one.

- 1) Trace the figure on tracing paper or any other material, which can enable you to see through (Figure 3.32).

**Figure 3.32**

- 2) Move the tracing paper on extended line (dotted line) until O reaches P or if you do not have a tracing paper, measure OP and then measure from P to get O' and P' (Figure 3.33).
- 3) Final position of triangle ABC will be triangle A'B'C' as indicated in Figure 3.33.

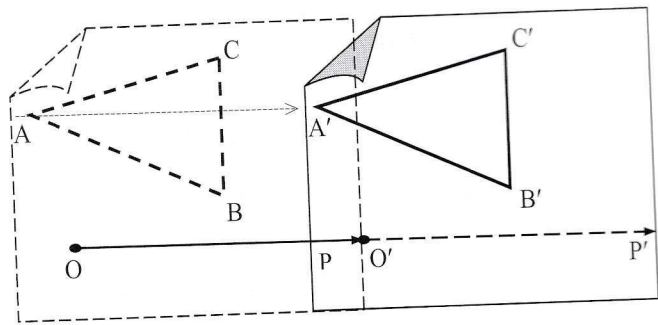


Figure 3.33

Now after you have studied the examples above you can do many other exercises on your own by drawing various figures and make translation on that figure, remember this need more practice. I advise you to do more practice before you do the following exercise.



## Activity 3

- In Figure 61 below, there is a plane made up of points and various figures indicated within it. In each part draw the image of the indicated figure under translation AB.

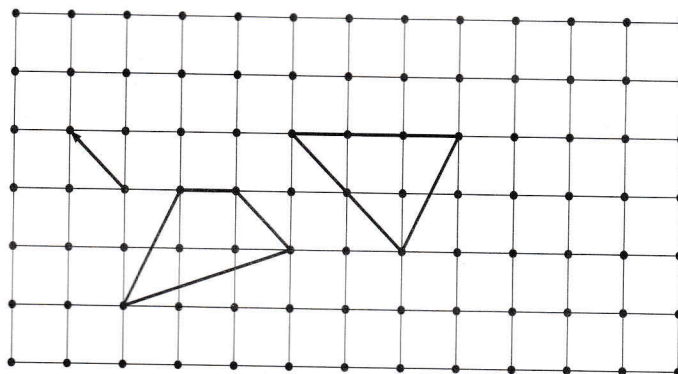
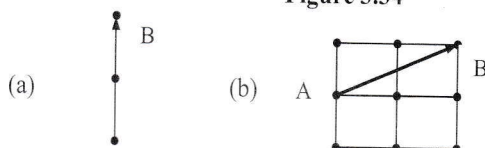


Figure 3.34



Compare your answers with key answers at the end of the unit.

## Enlargement

Dear learner, what about enlargement? Enlargement (scaling transformation or magnification) is a transformation that stretches or "shrinks" the plane away from or towards a specific point. Consider the case of map of Tanzania, which is drawn on a piece of paper, this map can be drawn on a big sheet of paper but still giving the same map.

The concept of enlargement in mathematics is dealt with when dealing with a similar figures. Take the case of a common figure, such as triangle. Here are similarity properties of triangles; two triangles are similar if:

- Two pairs of corresponding sides are **proportional** and their included angles are congruent. (SAS)

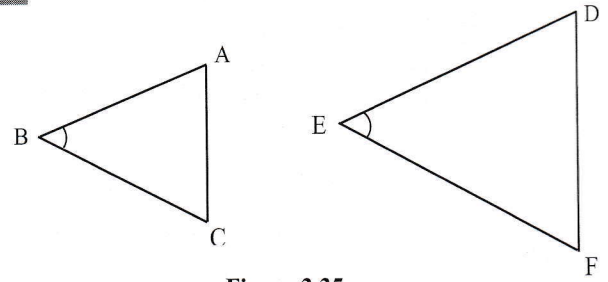


Figure 3.35

$$AB \sim DE$$

$$BC \sim EF$$

2. Two pairs of corresponding angles are **congruent** (AA).

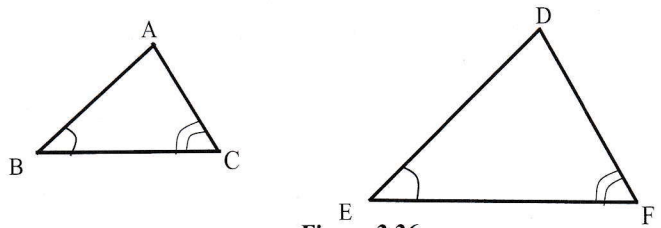


Figure 3.36

$$\angle ABC = \angle DEF$$

$$\angle ACB = \angle DFE$$

3. All three pairs of corresponding sides are proportional (SSS).

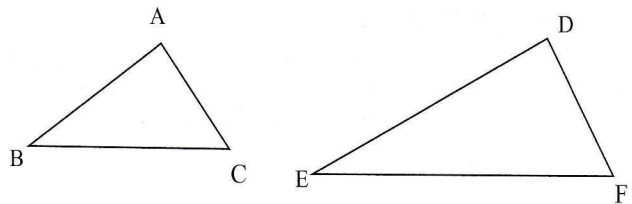


Figure 3.37

$$AB \sim DE$$

$$AC \sim DF$$

$$BC \sim EF$$

The indicated figures of triangles in Figures 3.36, 3.37 and 3.38 are not equal triangles, they are similar, this means that one is enlargement of the other.



It is not only triangle which can be enlarged or reduced in size, but because we cannot use the properties of triangle for similarity we use the general case for enlargement or reduction in size of figure.

The general technique is by the use of a scaling factor.

### Scaling Factor

Scaling factor is a technique used in enlargement or reduction of figures. In scaling factor the relationship between two figures is that, the corresponding side of two figures is related by a factor  $k$ .

For enlargement  $k > 1$ , for reduction  $k < 1$  but  $k$  is always positive.

#### For example

Let  $OAB$  be any triangle

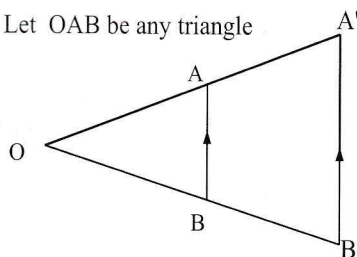


Figure 3.39

By scale factor  $k$  we can extend our triangle to have triangle  $OA'B'$ , by drawing line  $A'B'$  parallel to  $AB$ .

Then triangle  $OAB$  is similar to triangle  $OA'B'$ .

In enlargement: triangle  $OA'B'$  is enlargement of triangle  $OAB$ .

This can be proved as follows.

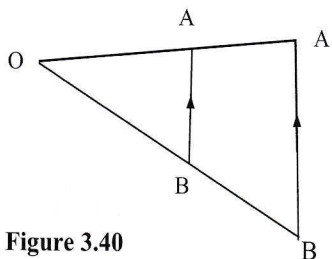


Figure 3.40

From Figure 3.40;

$$m(\angle AOB) = m(\angle A'OB') \text{ and } OA' = k(OA)$$

$$OB' = k(OB) \text{ Therefore}$$

$\triangle OAB \sim \triangle OA'B'$  by SAS (Side, Angle between and Side are proportional) and

$$\therefore A'B' = k(AB)$$



$\therefore$  Triangle  $OA'B'$  is obtained by multiplying the corresponding side of triangle  $OAB$  by a factor  $k$ .

#### Some examples of scaling factor

1:2, This means that  $k = 2$ , and therefore the corresponding sides of a given figure must be multiplied by 2 to get the required actual figure.

#### Example

Let  $ABCD$  be a  $2\text{cm} \times 2\text{cm}$  square (Figure 3.41), by a scale 1:2 we magnify.

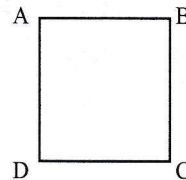


Figure 3.41

Our aim is to get figure  $A'B'C'D'$  (Figure 3.42), with  $4\text{cm} \times 4\text{cm}$  square

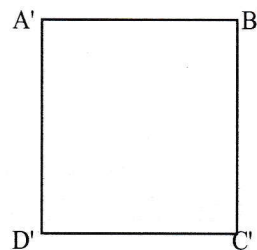


Figure 3.42

Another example of scale:

$1:\frac{1}{2}$ ; this means  $k = \frac{1}{2}$ , and therefore the corresponding sides of a given figure must be multiplied by  $\frac{1}{2}$  to get the required figure.

From Figure 3.42 or square is reduced in size to  $1\text{cm} \times 1\text{cm}$  square,  $A''B''C''D''$  (Figure 3.43).

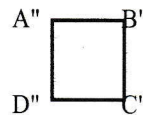


Figure 3.43



### Common Techniques used in Enlargement

Dear learner, let us now look at common techniques used in enlargement

#### By using squares

Figures, especially irregular figures, are reduced or enlarged by use of squares. Irregular figures means any figure which its corresponding sides are not straight or a figure which cannot be named by common name as a square, rectangle, triangle etc. This includes your picture, maps of countries etc.

**For example,** figure 3.44 is an irregular figure drawn in  $1\text{cm} \times 1\text{cm}$  square, to enlarge it by factor of 2 ( $k = 2$ ) we draw it in  $2\text{cm} \times 2\text{cm}$  square Figure 3.45.

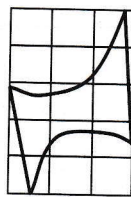


Figure 3.44

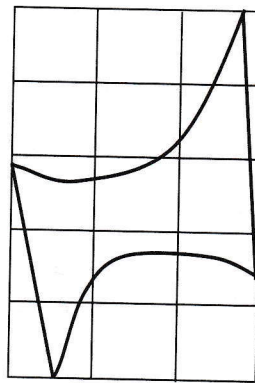


Figure 3.45

#### Radial techniques

If you are given a figure and you want to magnify (enlarge) or reduce the size of it, then use Radial techniques. This is more useful on figures with sides which are straight lines.

**For example,** figure RSTU is a trapezium enlarged by a scaling factor of 2 ( $k = 2$ ) (Figure 3.46).

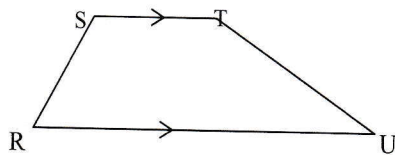
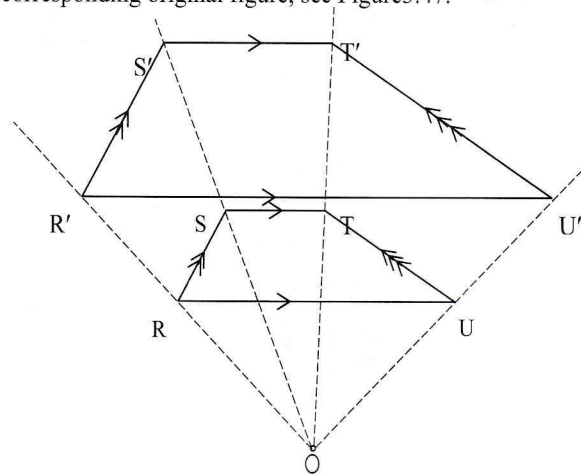


Figure 3.46

**Solution**

Draw your figure 3.46 and select your centre inside or outside the figure then make arrows starting from centre  $O$  to the four corners; for construction use dotted lines. Extend your dotted line as far as you can then draw required figure with your line parallel to the corresponding original figure, see Figure 3.47.

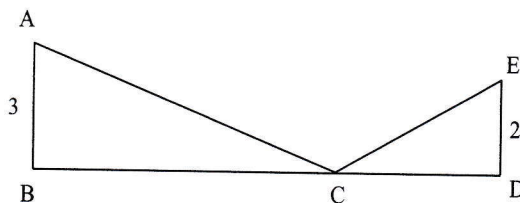
**Figure 3.47**

$R'S'T'U'$  is the required figure.

**Activity 4**

Discuss the following questions

1. Draw any triangle and draw its image by factor  $k = \frac{1}{2}$ .
2. A circle is having a radius of 2cm, draw its image by a factor  $k = 2$ .
3. Given that triangle ABC Figure 3.48 is similar to triangle EDC, what is the scale factor of the enlargement?



**Figure 3.48**

4. Draw any figure and magnify it using a grid (square) method by scale factor of 3.
5. Draw the image of a square by scale factor of  $1\frac{1}{2}$  and the centre being at the centre of your square by radial techniques.

*Compare your answers with those given at the end of the unit.*

## Combined Transformation, Reflection and Translation

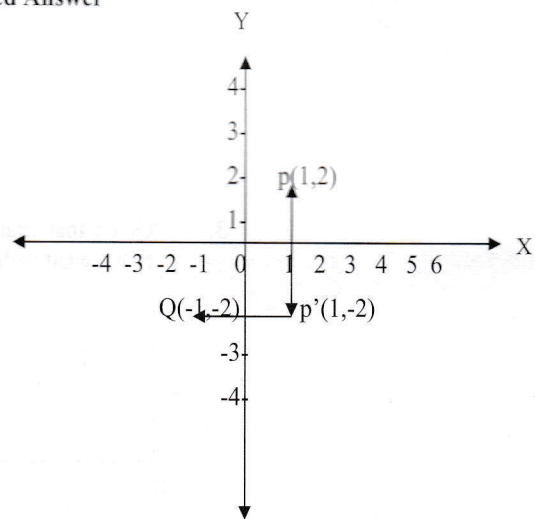
**Activity 5**

Reflect point P(1,2) on the X-axis. Then reflect its image of P on Y-axis.

If the found image is Q show its location in the x-y plane.



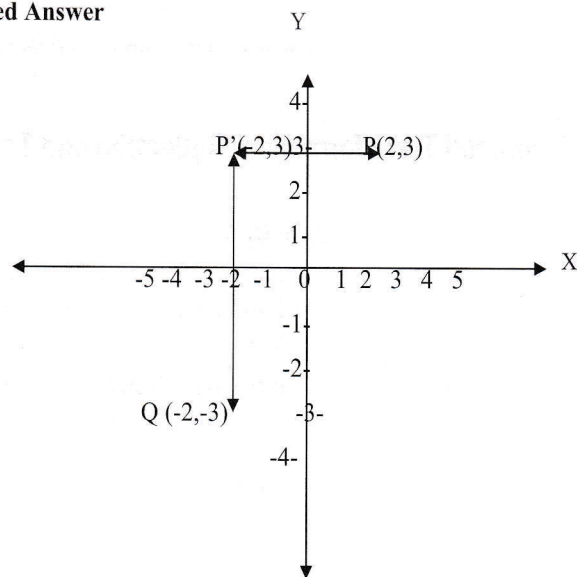
Expected Answer

The image Q will be at  $(-1,-2)$ 

## Activity 6

Reflect the point  $p(2,3)$  on  $y$ -axis followed by reflection on  $x$ -axis.

Expected Answer





### Activity 7

1. Reflect the point  $(1,3)$  on the line  $x=y$  followed by reflection on line  $x$ -axis.

Expected answer  $(3,-1)$ .

2. The point  $Q(2,4)$  is translated under  $(4,0)$ . Determine its image after being reflected in the  $x$ -axis.

**Step 1:** Determine the coordinates of  $Q$  after translated

Expected:  $(2,4) + (4,0) = (6,4)$ .

**Step 2:** Reflect  $(6,4)$  on the  $x$ -axis.

Expected  $M_x(6,4) = (6,4)$

### Activity 8

1. The translation which maps the point  $p(1,-2)$  on to the point  $p(3,4)$  also maps the point  $Q(9,6)$  on to the point  $Q'(2,5)$ . Find the coordinates of  $Q$ . Expected  $Q(-2,-1)$ .

2. The point  $p(-2,3)$  is reflected on the  $ex$ -axis and then its image is translated by  $(2,3)$ . Find the coordinator of the image after been translated. Expected answer  $(0,0)$ .

## Reflection and Rotation

In this section you will learn how to reflect a point in a given axis followed by rotation.

### Activity 9

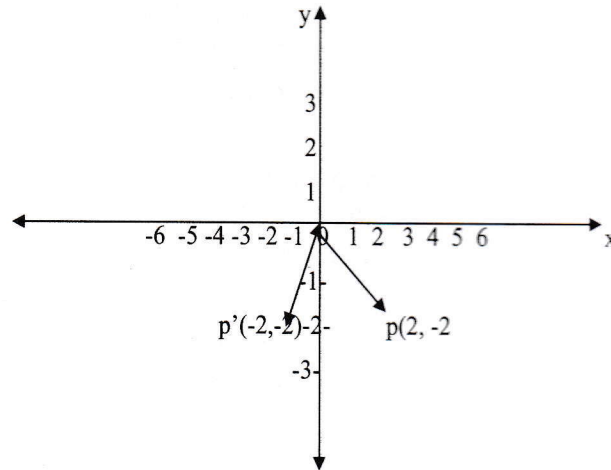
You are given point  $p(2,2)$  on the  $x$ - $y$  plane:

- (i) Reflect the point  $p$  on  $x$ -axis
- (ii) What is the image of point  $p$ ?
- (iii) Rotate the image of  $p$  through angle of  $90^\circ$  clockwise
- (iv) What are the coordinates of the image of  $p$  after notated?

### Expected Answers

Step 1: Draw the  $x$ - $y$  plane

Step 2: Indicate the translation

**Activity 10**

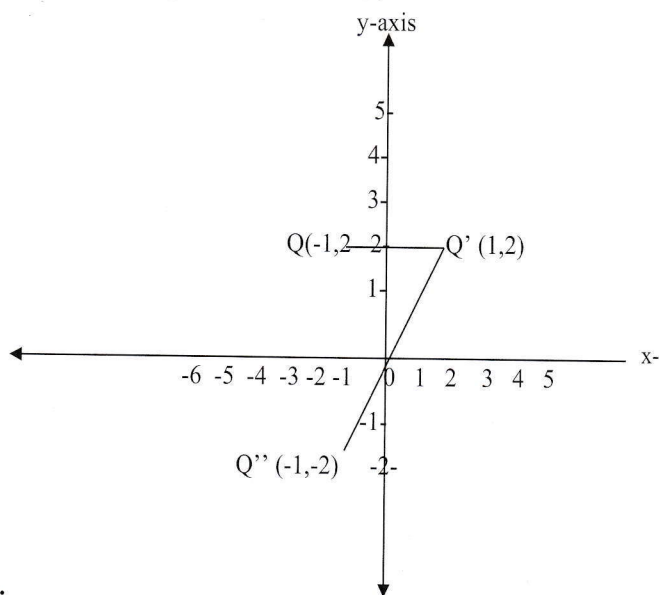
Reflect the point  $(-1, 2)$  on the  $y$ -axis, then rotate the image  $Q$  through angle of  $180^\circ$  respectively. What is the coordinate of  $Q''$ ?

**Expected Answers**

- Step 1:** (i) Draw the  $x$ - $y$  axes.
- (ii) Indicate the point  $Q(-1, 2)$  on the  $x$ - $y$  axes
- (iii) Reflect the point  $(1, -4)$  on the  $y$ -axis.
- (iv) Rotate the image of the point  $Q$  after reflection through an angle  $180^\circ$ .
- (v) Give the coordinate of the final image.



## Expected x-y plane



## Step 2:

Draw conclusion.

That  $M_s (-1, 2) = (1, 2)$

and  $R_{180} (1, 2) = (-1, -2)$



## Unit Reflection



Dear learner, you have now completed this unit, can you remember the things that you have ever used in daily life that may relate with the concept of rotation and reflection?

Imagine you are riding a motorcycle and you are about to turn right at the corner, can you determine the distance of the car coming behind you by using the sight mirror?

Imagine also you have parked your car in the wrong parking and you need to move it to the right parking. Can you relate the concept of translation with change of car position from the wrong parking to the right parking?

Similarly a room has a bed of 3 x 6ft that can accommodate two children. Two more adults need to be accommodated in the same room. What size of the bed will fit them?



## Unit Assignment



1. Find the image of point  $A(2,1)$  after reflection in the  $x$ -axis.
2. Find the image of  $B(3,4)$  under a reflection in the  $y$ -axis.
3. Find the image of the point  $A(1,2)$  after reflection in the true  $y=x$ .
4. Find the image of  $B(3,4)$  after a reflection in the line  $y=-x$ .
5. Find the image of the point  $(1,2)$  under the rotation through  $180^\circ$  in the anticlockwise direction.
6. Find the image of  $(3,5)$  after rotation of  $90^\circ$  about the origin  $h$ .
7. Find the image of  $(3,5)$  after rotation of  $270^\circ$  about the origin  $h$ .
8. If the point  $(3, 5)$  under a given line of reflection is  $(-3,5)$  find the line of reflection.
9. If the image of  $p$  under a reflection on the  $y = 0$  is  $(-4,3)$ , find the object.
10. Draw  $DABC$  and its image  $DA'B'C'$  after a reflection in the line  $y-x$  of  $A(0,3)$   $B(3,0)$   $C(3,2)$ . What is this line of symmetry of the two figures (s).
11. Find the image of  $(3,4)$  after rotation of  $-90^\circ$  about the any  $h$ .
12. Find the image of  $(3,3)$  after rotation of  $45^\circ$  about the origin.
13. If the point  $(5,2)$  under a given line of reflection is  $(-5, 2)$ . Find the line of reflection.
14. Construct the enlargement of the triangle  $ABC$  with your centre at  $O$  using radial techniques,  $k = 2$ .

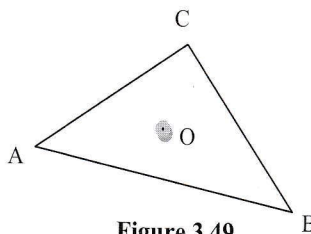


Figure 3.49



15. Plot the triangle ABC with A(2, 3) B(-1, 4) and C(-2, -1) in the  $x - y$  plane. Draw the image of the triangle and  $T(x, y) = (y, x)$ .
16. Graph triangle  $\triangle ABC$  with A(3, 1), B(4, 3) and C(5, -2) and its image under reflection  $M_l$  where  $l$  is the line  $x = 0$ .
17. A square of vertices A(0,0) B(0,1) C(1, 1) and D(1, 0) is in  $x-y$  plane. Find its image under enlargement of factor of 3 ( $k = 3$ ).
18. The point A(5,-4) is translated by (2,3), followed by reflection in the  $y = 0$ . Find the image of A.

### Key Answers for Activities

#### Activity 1

1.

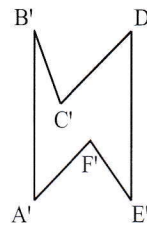
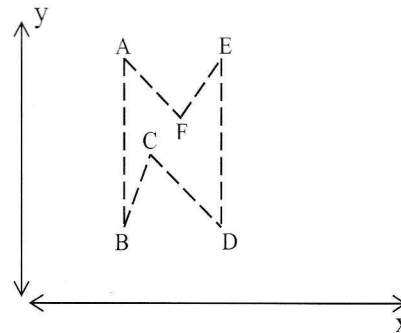
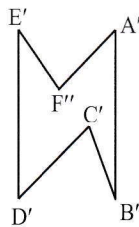
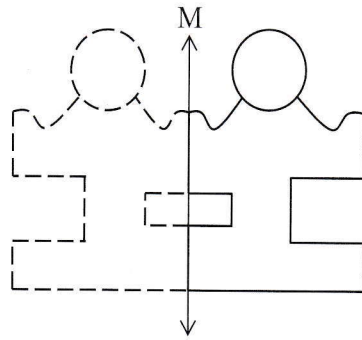


Figure 4.50



2.



Image

Figure 4.51

3. (a) (3, 1) (b) (-6, 3)

4.

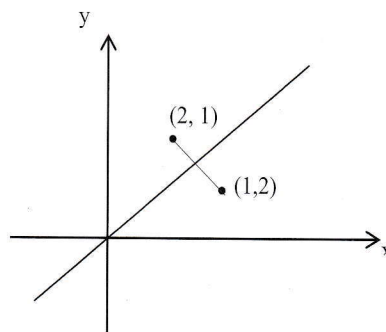


Figure 3.52

Activity 2

1.

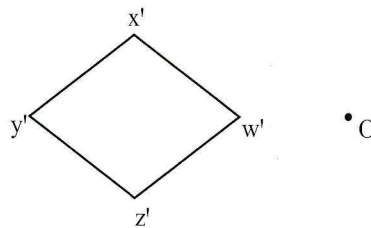


Figure 3.53

2. (a) (-3, 1) (b) (4, 3) (c) (4, -2)



3. (a)

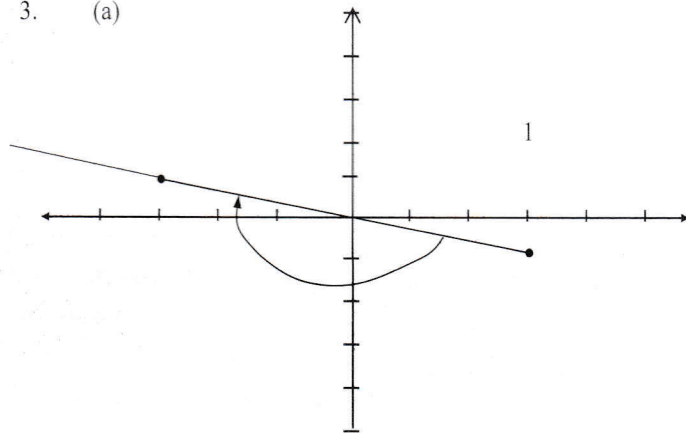


Figure 3.54

Activity 3

1 (a)

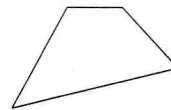


Figure 3.55

(b)

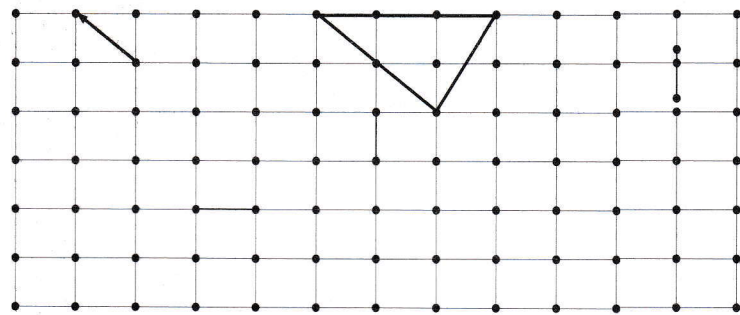


Figure 3.56



Activity 4

1.

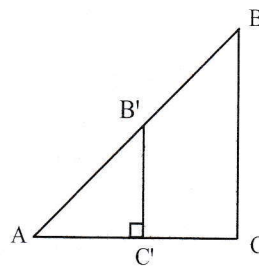


Figure 3.57

$\triangle AB'C'$  is the image  $\triangle ABC$

2. Draw a circle of radius 2cm and then using the same centre O draw a circle of radius 4cm.

3.  $\frac{3}{2}$

4.

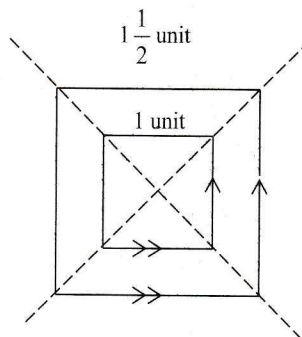


Figure 3.58



## References

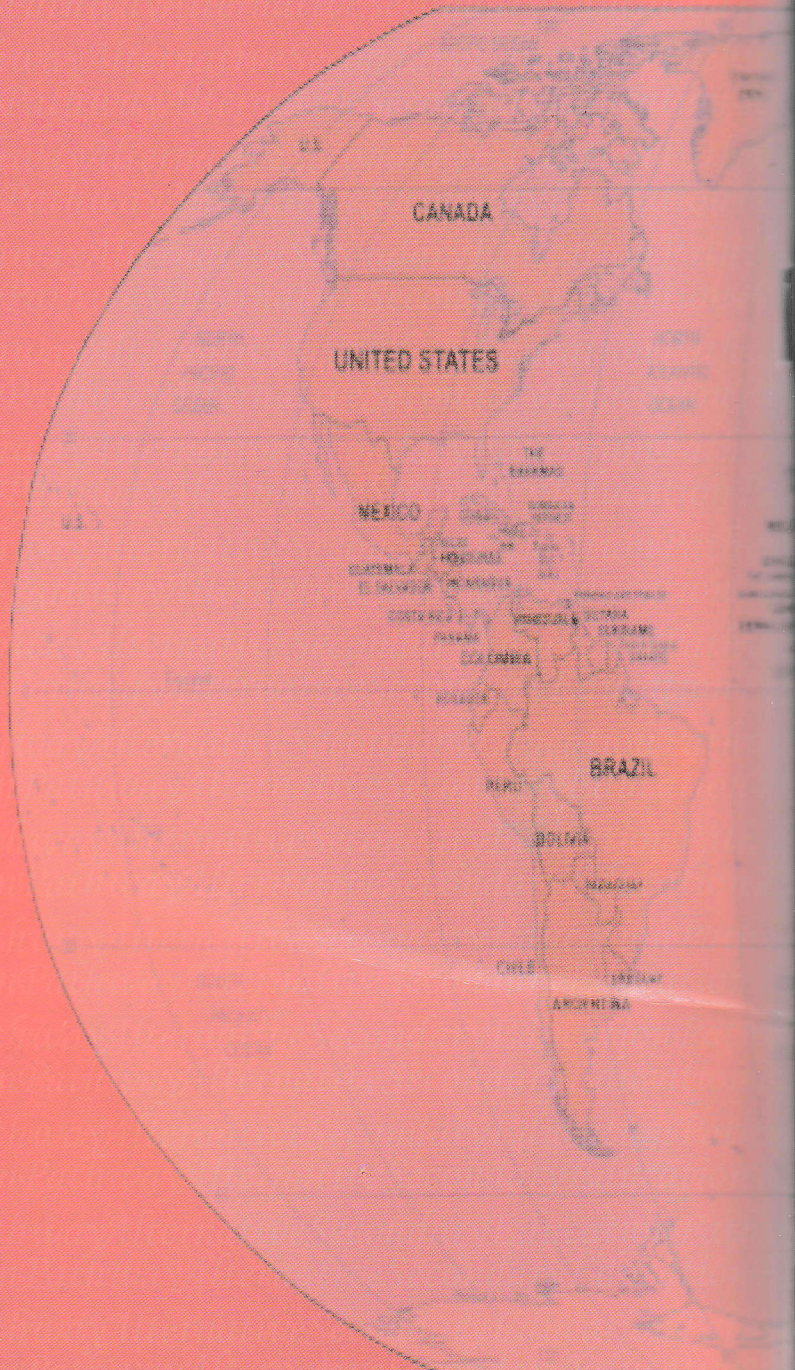
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